Slide #

A CPS1-driven market for Clearing/Settling Inadvertent Interchange

Simulation from an 11-Day period of 17-control-area Western-Interconnection Jan/’02 hourly data

presented to

Inadvertent Interchange Payback Taskforce

North American Energy Standards Board

by Robert Blohm

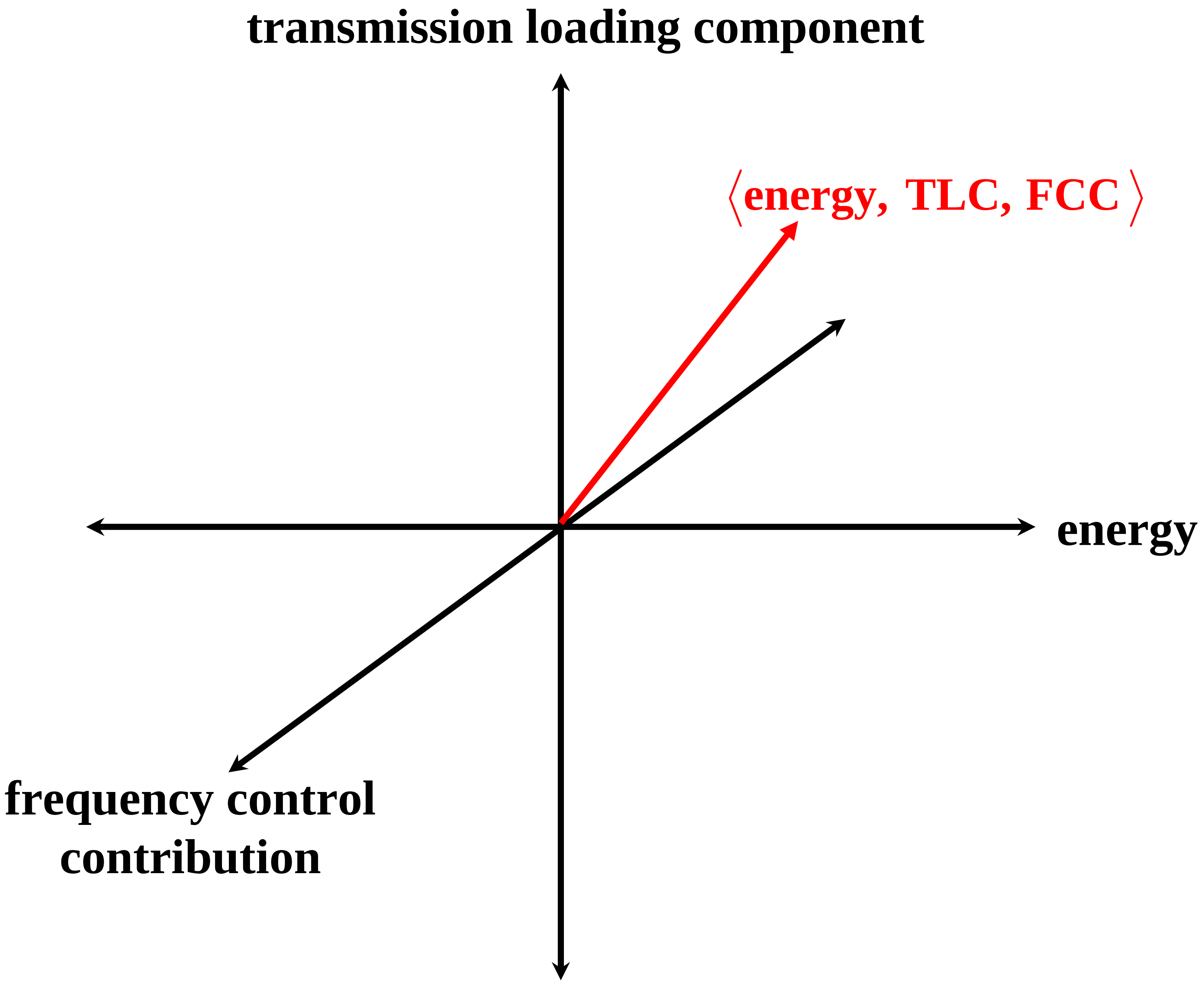
Houston

December 10, 2003

Inadvertent Interchange is more than just energy
at the price of scheduled energy.

It's also unscheduled contribution or correction to frequency error.

Inadvertent is a vector in a state space



Dual pricing of unscheduled energy

Ambiguity along the diagonal.
Diagonal occurs only when frequency is high.
Off-diagonals occur only when frequency is low.

Revenue/Expense

Unscheduled part $\beta > 0$ / energy part $U > 0$	good	bad
sold	$p_e U + p_\beta 10\beta$	$p_e U - p_\beta 10\beta$
bought	$-p_e U + p_\beta 10\beta$	$-p_e U - p_\beta 10\beta$

receive

pay

Four possible combinations of
Priced Energy Component
and
Priced
Frequency Control Contribution
of Inadvertent Interchange

Case 1: Frequency High and Control Area A "Leaning" on Rest of Interconnection for 200 MWh

1. Energy Component:

A pays the Interconnection $10/MWh energy price for 200 MWh = -$ 2000

2. Frequency Control Contribution:

A receives from the Interconnection $15* of FCC_p per MWh of Inadvertent = +$ 3000 for 200 MWh of Inadvertent Interchange that is opposite to the frequency error.

*FCC_p per MWh would be less than $15 if the average frequency error were smaller.

3. Net Result:

A receives from the Interconnection a net total of $ 1000*.

*The net result could be a payment if the average frequency error is small enough.

Case 2: Frequency High and Rest of Interconnection "Leaning" on Control Area A for 200 MWh

1. Energy Component:

A receives from the Interconnection $10/MWh energy price for 200 MWh = +$ 2000

2. Frequency Control Contribution:

A pays the Interconnection $15* of FCC_p per MWh of Inadvertent = -$ 3000 for 200 MWh of Inadvertent Interchange that is contributing to the frequency error.

*FCC_p per MWh would be less than $15 if the average frequency error were smaller.

3. Net Result:

A pays the Interconnection a net total of $ 1000*.

*The net result could be a receipt if the average frequency error is small enough.

Case 3: Frequency Low and Control Area A "Leaning" on Rest of Interconnection for 200 MWh

1. Energy Component:

A pays the Interconnection \$20/MWh energy price for 200 MWh = -\$ 4000

2. Frequency Control Contribution:

A pays the Interconnection \$15 of FCC_p per MWh of Inadvertent = -\$ 3000 for 200 MWh of Inadvertent Interchange that is contributing to the frequency error.

3. Net Result:

A pays the Interconnection a combined total of -\$ 7000.

Case 4: Frequency Low and Rest of Interconnection “Leaning” on Control Area A for 200 MWh

1. Energy Component:

A receives from the Interconnection \$20/MWh energy price for 200 MWh = +\$ 4000

2. Frequency Control Contribution:

A receives from the Interconnection \$15 of FCC_p per MWh of Inadvertent = +\$ 3000 for 200 MWh of Inadvertent Interchange that is opposite to the frequency error.

3. Net Result:

A receives from the Interconnection a combined total of +\$ 7000.

Summary of the 4 cases

"Leaning" by: \ Frequency:	High	Low
Control Area A	Case 1 A receives $1000	Case 3 A pays $7000
Rest of Interconnection	Case 2 A pays $1000	Case 4 A receives $7000

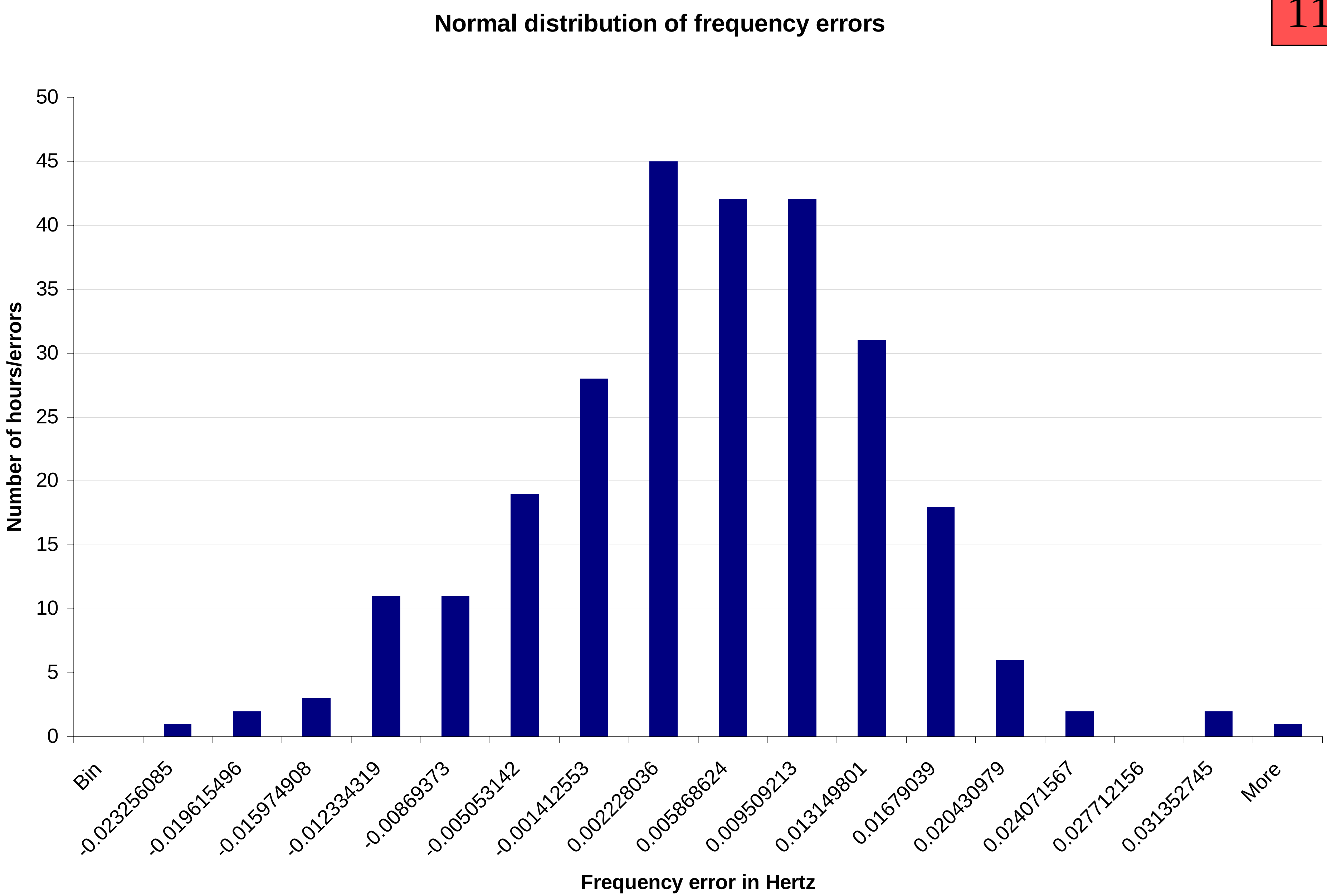
Animation of the Clearing/Settlement

of

Frequency Control Contribution

based on data of an 11-day period in January 2002

on 17 control-area Western Interconnection:


Normal distribution of frequency errors
Number of hours/errors
50
45
40
35
30
25
20
15
10
5
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-0.023256085
-0.019615496
-0.015974908
-0.012334319
-0.00869373
-0.005053142
-0.001412553
0.002228036
0.005868624
0.009509213
0.013149801
0.01679039
0.020430979
0.024071567
0.027712156
0.031352745
More
Frequency error in Hertz

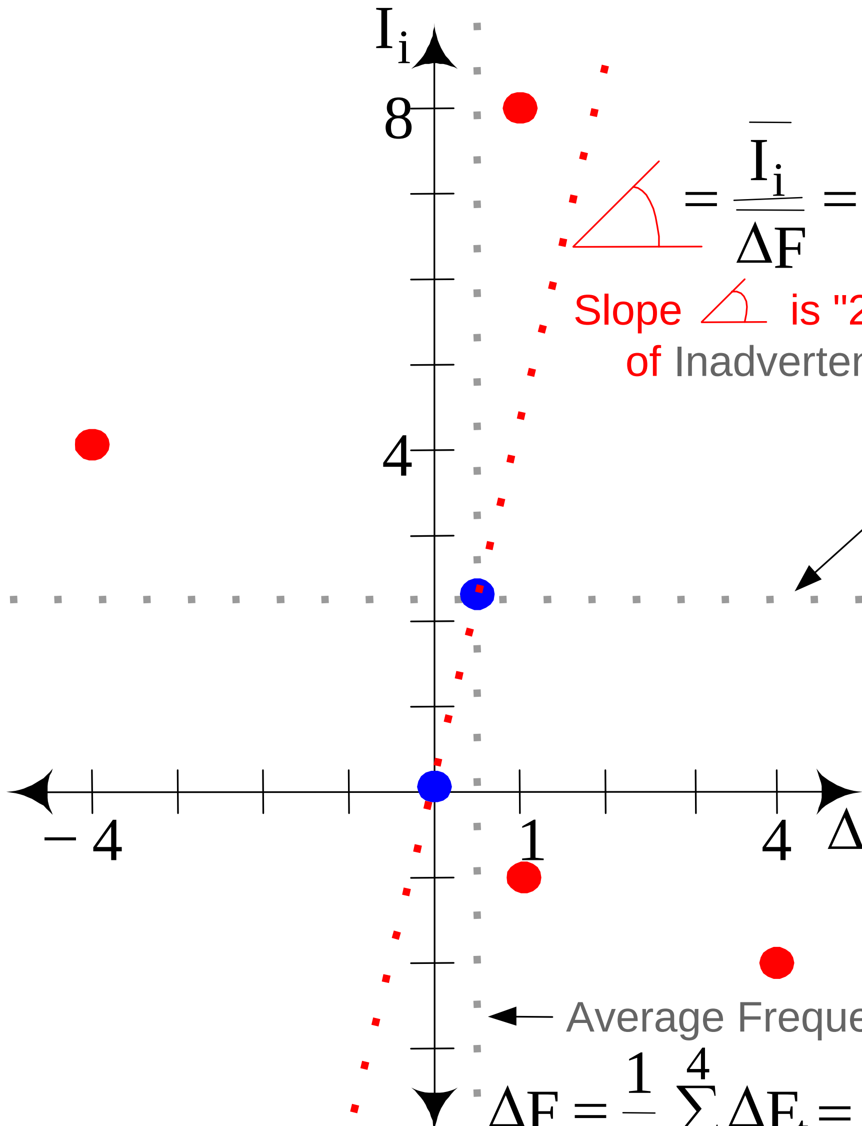
A Balancing Authority i's Frequency Control Contribution is a "2-dimensional average" of Inadvertent and Frequency-error each weighted by Frequency error.

A "2-dimensional average" is the slope of a line from the origin through the intersection of the lines intercepting the two averages.

4-period scatter of Balancing Authority i's <Frequency-error, Inadvertent> points

$$< \Delta F_t , I_{i,t} >$$

denoted by the 4 red dots



$$\angle = \frac{\overline{I_i}}{\overline{\Delta F}} = \frac{9}{2}$$

Slope is "2-dimensional average" of Inadvertent & Frequency-error

i's Average Inadvertent

$$I_i = \frac{1}{4}\sum_{t=1}^{4} I_{i,t} = \overline{I_{i,t}} = \frac{9}{4}$$

Average Frequency-error

$$\Delta F = \frac{1}{4}\sum_{t=1}^{4} \Delta F_t = \overline{\Delta F_t} = \frac{1}{2}$$

Drawing prepared by Robert Blohm July 5, 2003

Period t	ΔF_t	$I_{i,t}$
1	4	-2
2	1	-1
3	-4	4
4	1	8
Sum =	2	9

A Balancing Authority i's Frequency Control Contribution is a "2-dimensional average" of Inadvertent and Frequency-error each weighted by Frequency error.

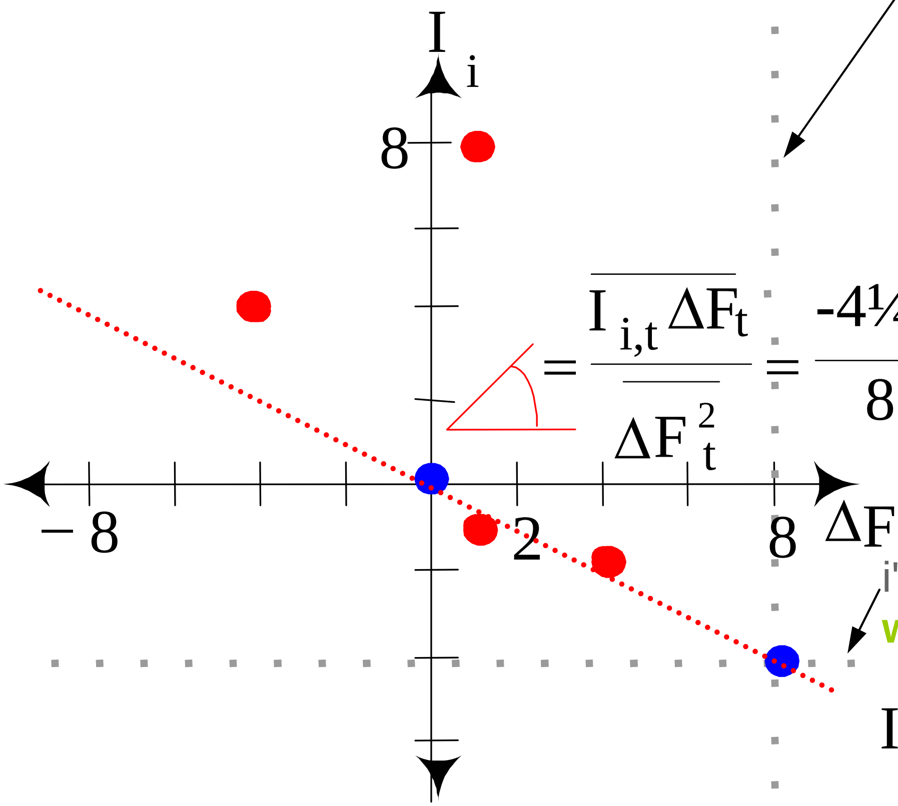
A "2-dimensional average" is the slope of a line from the origin through the intersection of the lines intercepting the two averages.

Period t	ΔF_t	$I_{i,t}$	$\overline{I_{i,t}\Delta F_t}$	$\overline{\Delta F_t^2}$
1	4	-2	-8	16
2	1	-1	-1	1
3	-4	4	-16	16
4	1	8	8	1
Sum =	2	9	-17	32

4-period scatter of Balancing Authority i's <Frequency-error, Inadvertent> points

$$< \Delta F_t , I_{i,t} >$$

denoted by the 4 red dots



$$\angle = \frac{\overline{I_{i,t}\Delta F_t}}{\overline{\Delta F_t^2}} = \frac{-4¼}{8}$$

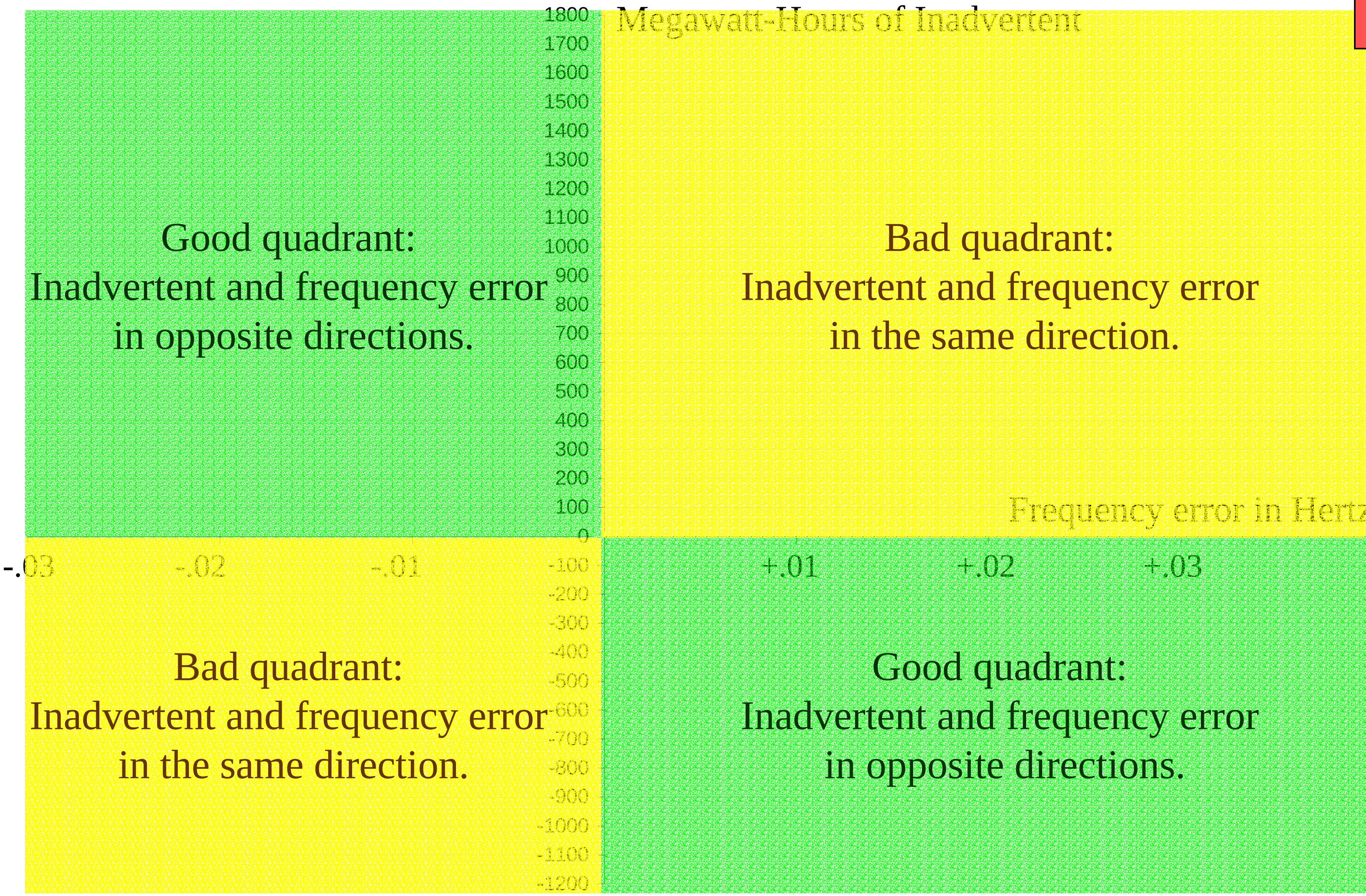
Slope is "2-dimensional average" of Inadvertent & Frequency-error **weighted by Frequency-error**

Average Frequency-error **weighted by Frequency-error**

$$\Delta F = \frac{1}{4}\sum_{t=1}^{4}(\Delta F_t \times \Delta F_t) = \overline{\Delta F_t^2} = 8$$

i's Average Inadvertent **weighted by Frequency-error**

$$I_i = \frac{1}{4}\sum_{t=1}^{4}(I_{i,t} \times \Delta F_t) = \overline{I_{i,t}\,\Delta F_t} = \frac{-17}{4} = -4¼$$


Megawatt-Hours of Inadvertent
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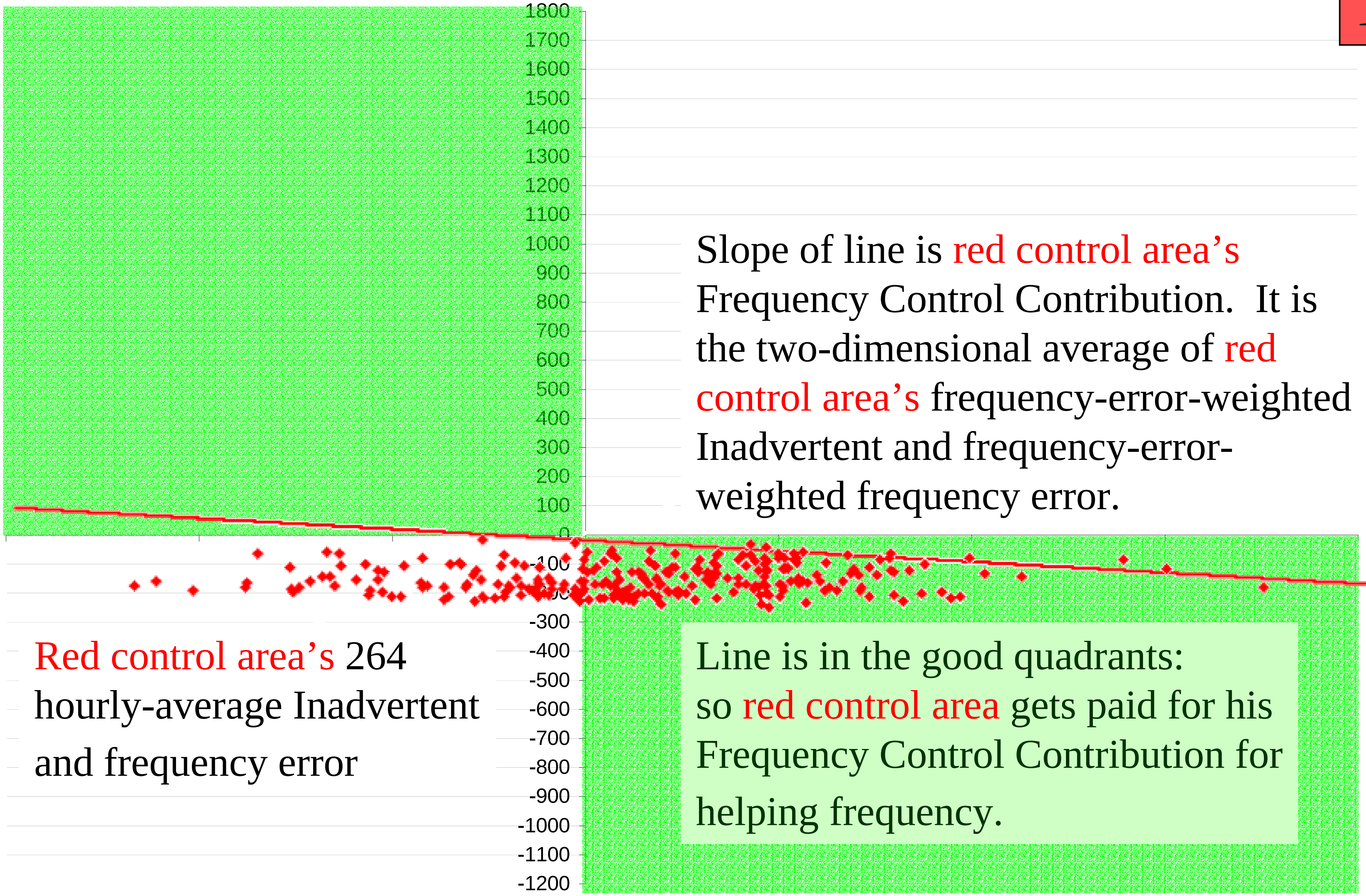
Frequency error in Hertz
-.03
-.02
-.01
+.01
+.02
+.03
Good quadrant:
Inadvertent and frequency error
in opposite directions.
Bad quadrant:
Inadvertent and frequency error
in the same direction.
Bad quadrant:
Inadvertent and frequency error
in the same direction.
Good quadrant:
Inadvertent and frequency error
in opposite directions.

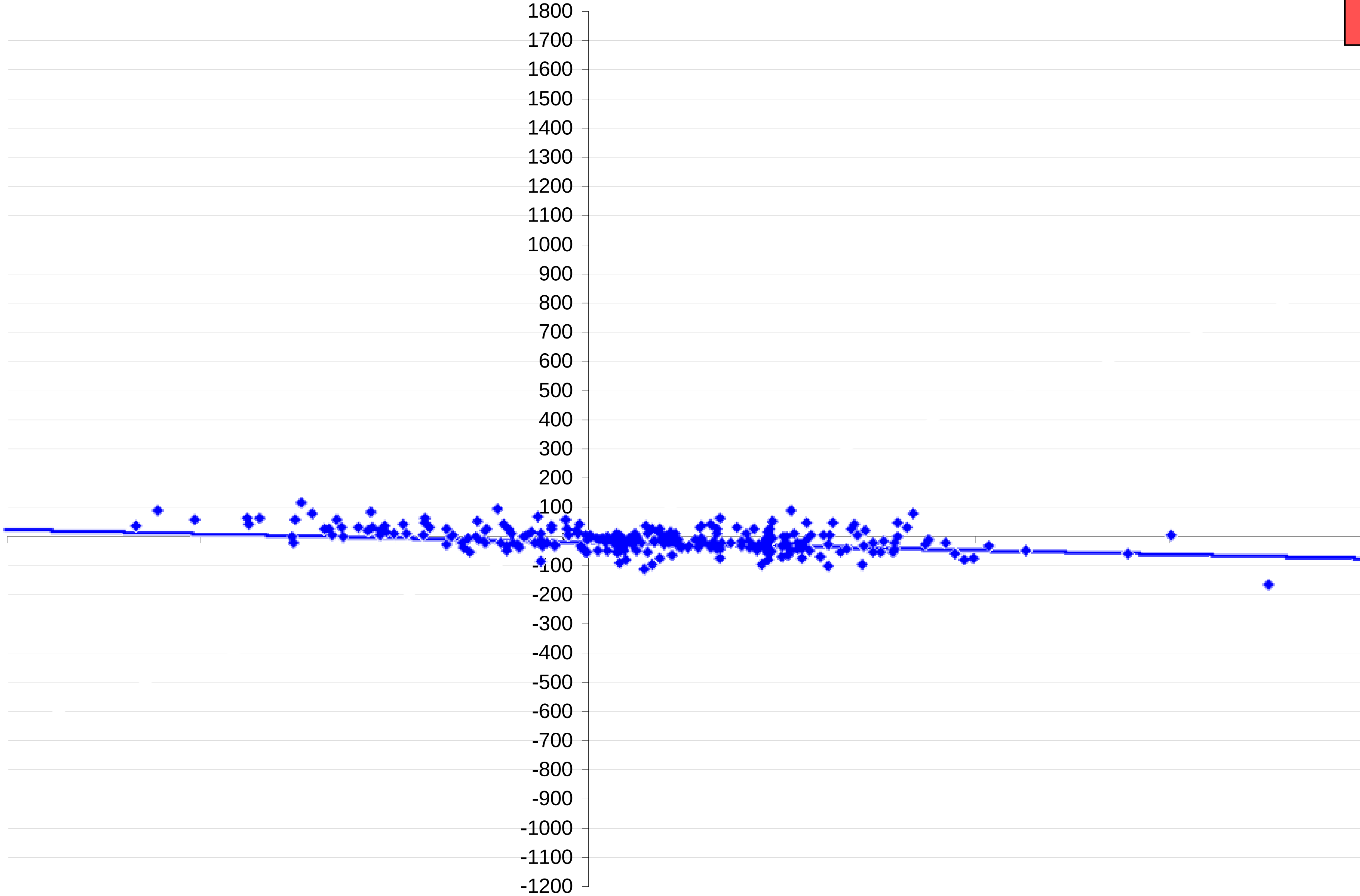


Slope of line is red control area’s Frequency Control Contribution. It is the two-dimensional average of red control area’s frequency-error-weighted Inadvertent and frequency-error-weighted frequency error.

Red control area’s 264 hourly-average Inadvertent and frequency error

Line is in the good quadrants: so red control area gets paid for his Frequency Control Contribution for helping frequency.

Control Area 1 (CA 1)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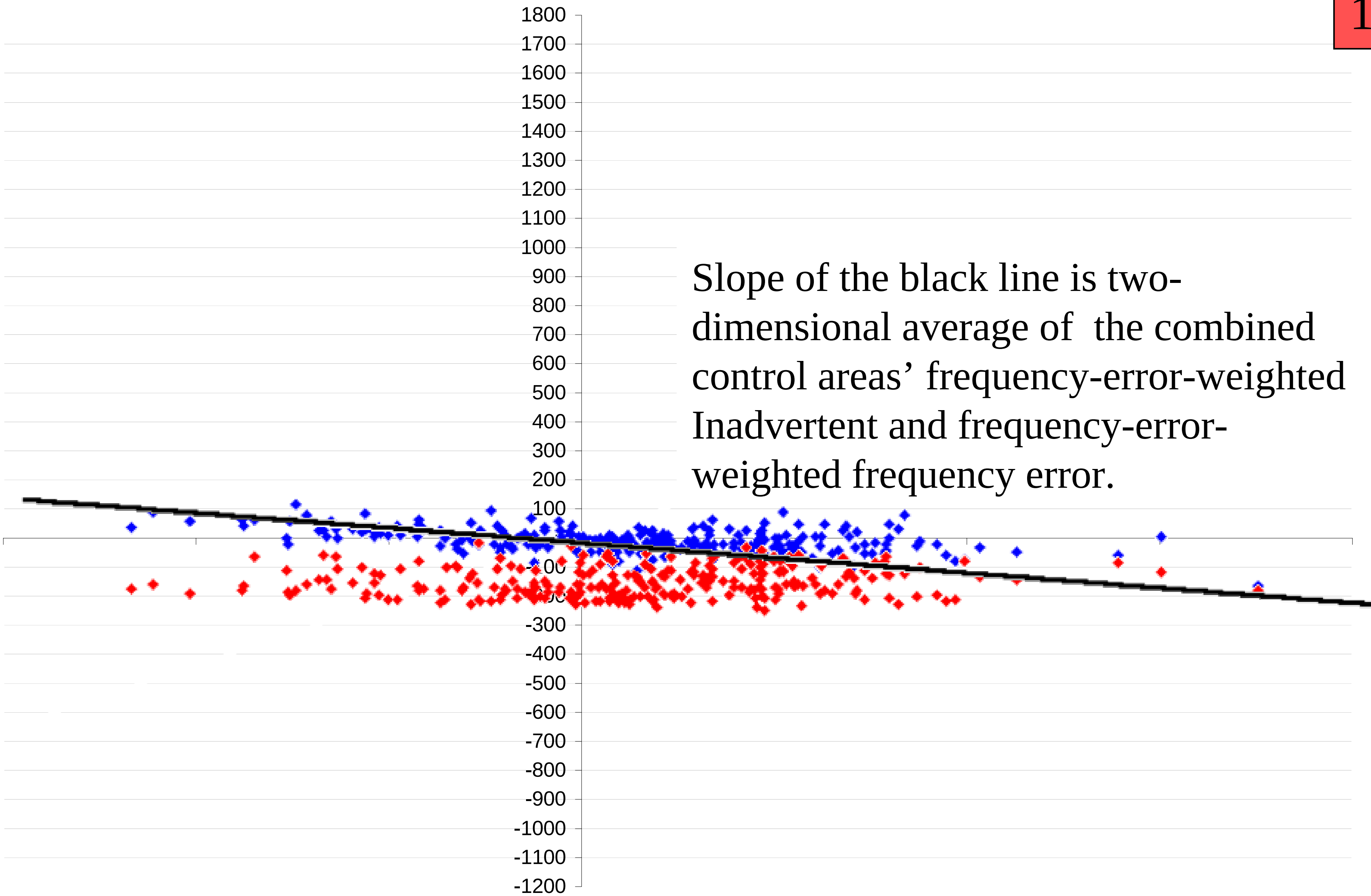
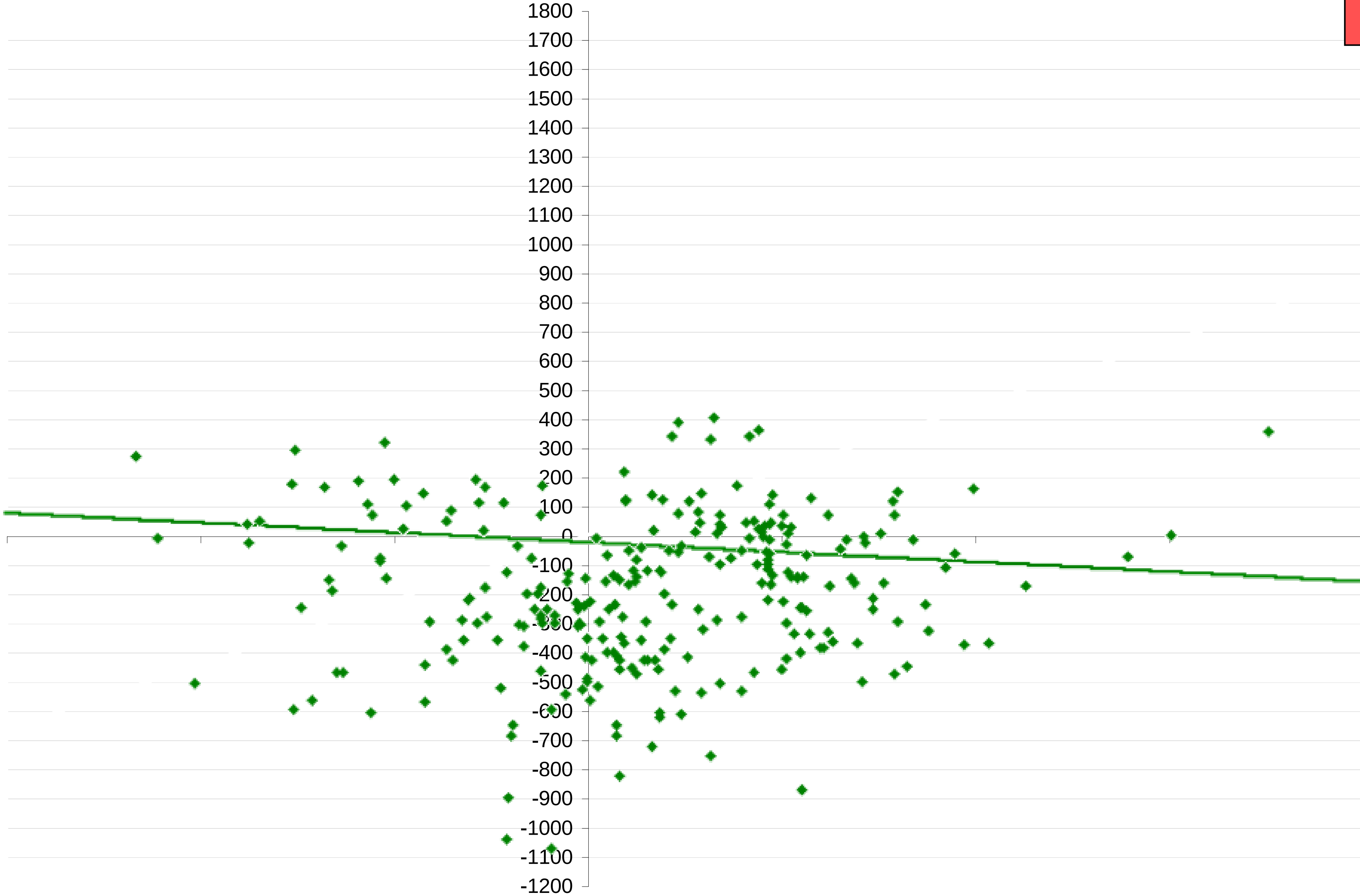
CA 2



Slope of the black line is two-dimensional average of the combined control areas' frequency-error-weighted Inadvertent and frequency-error-weighted frequency error.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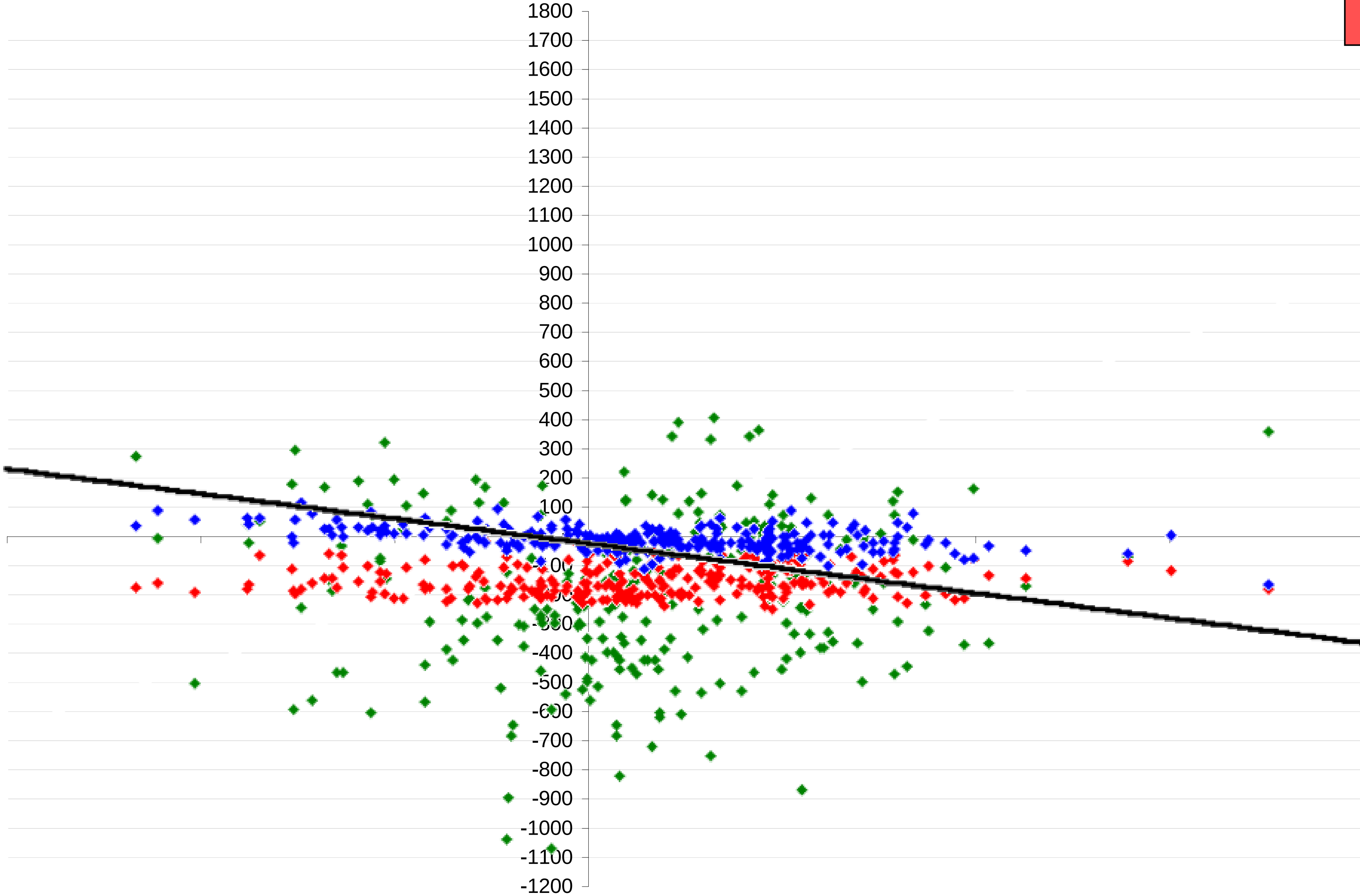
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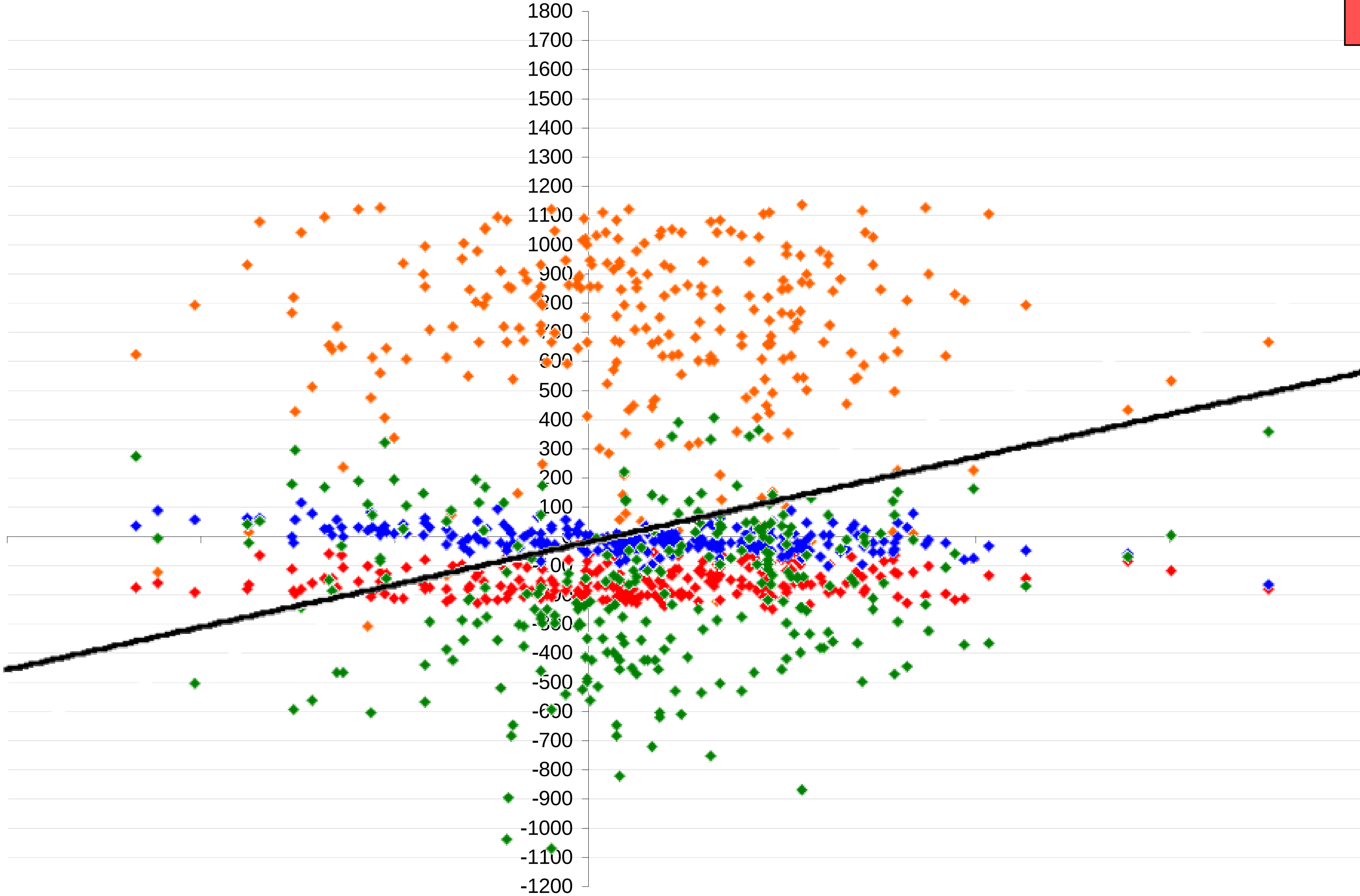
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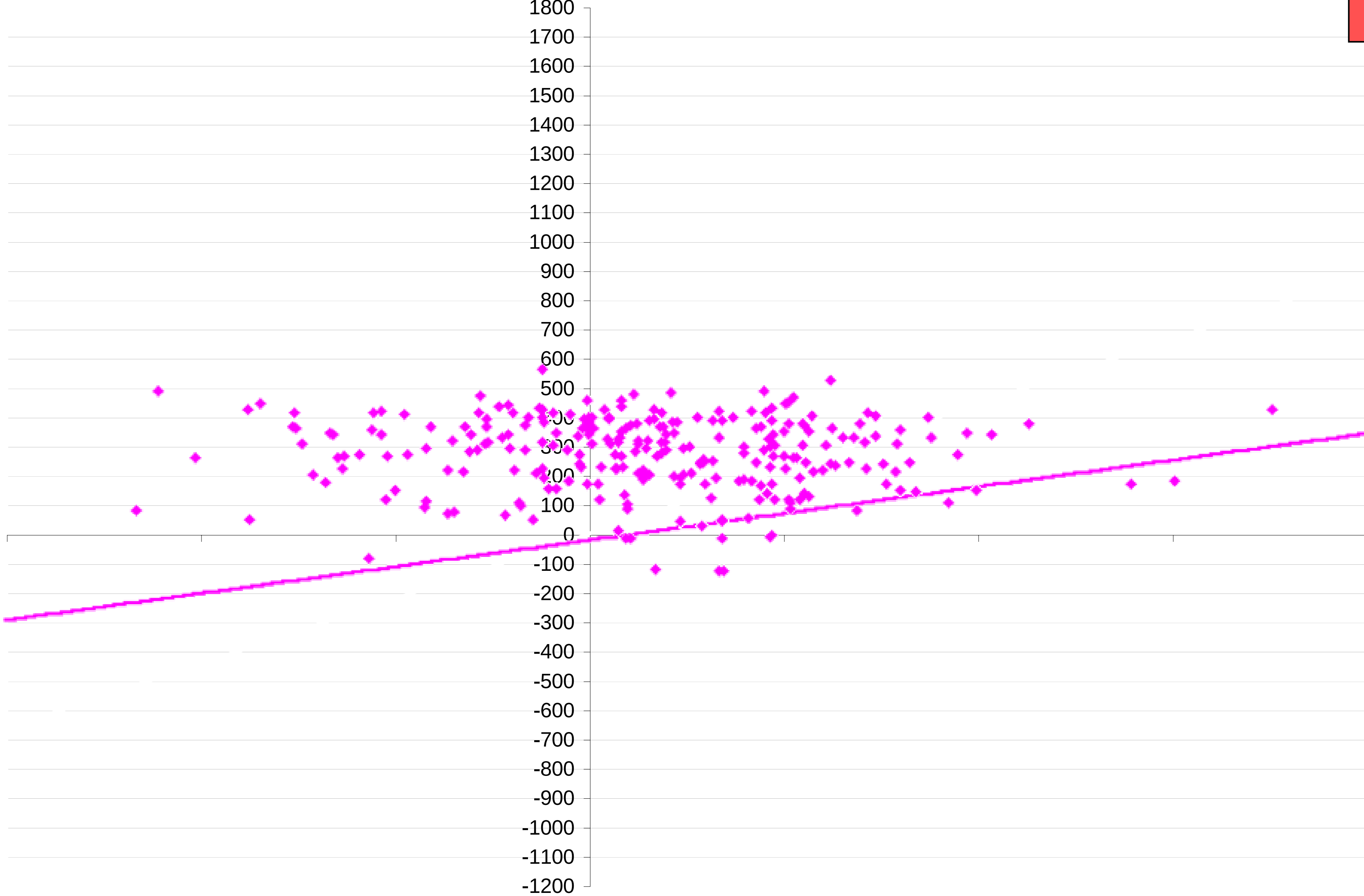
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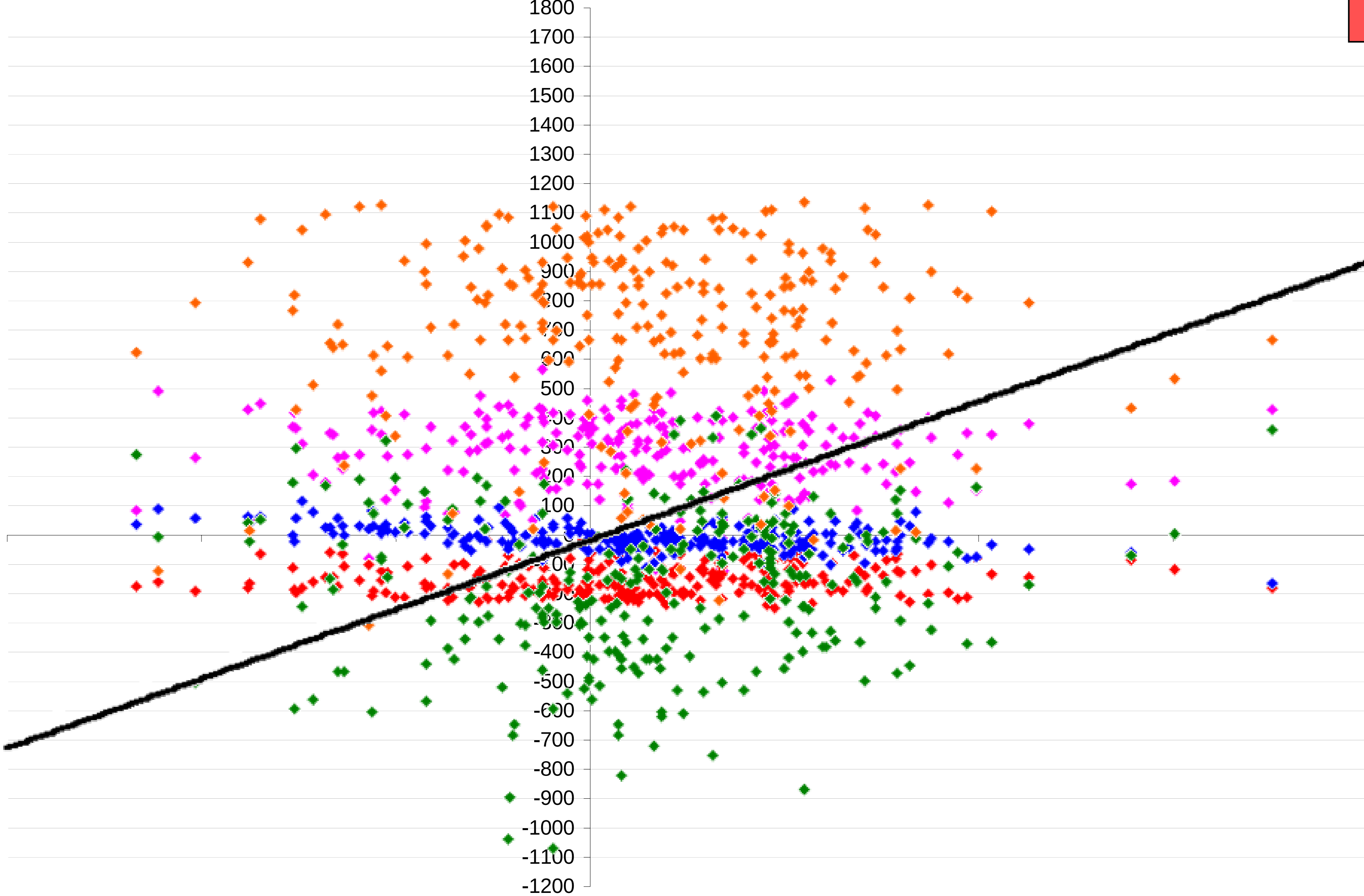
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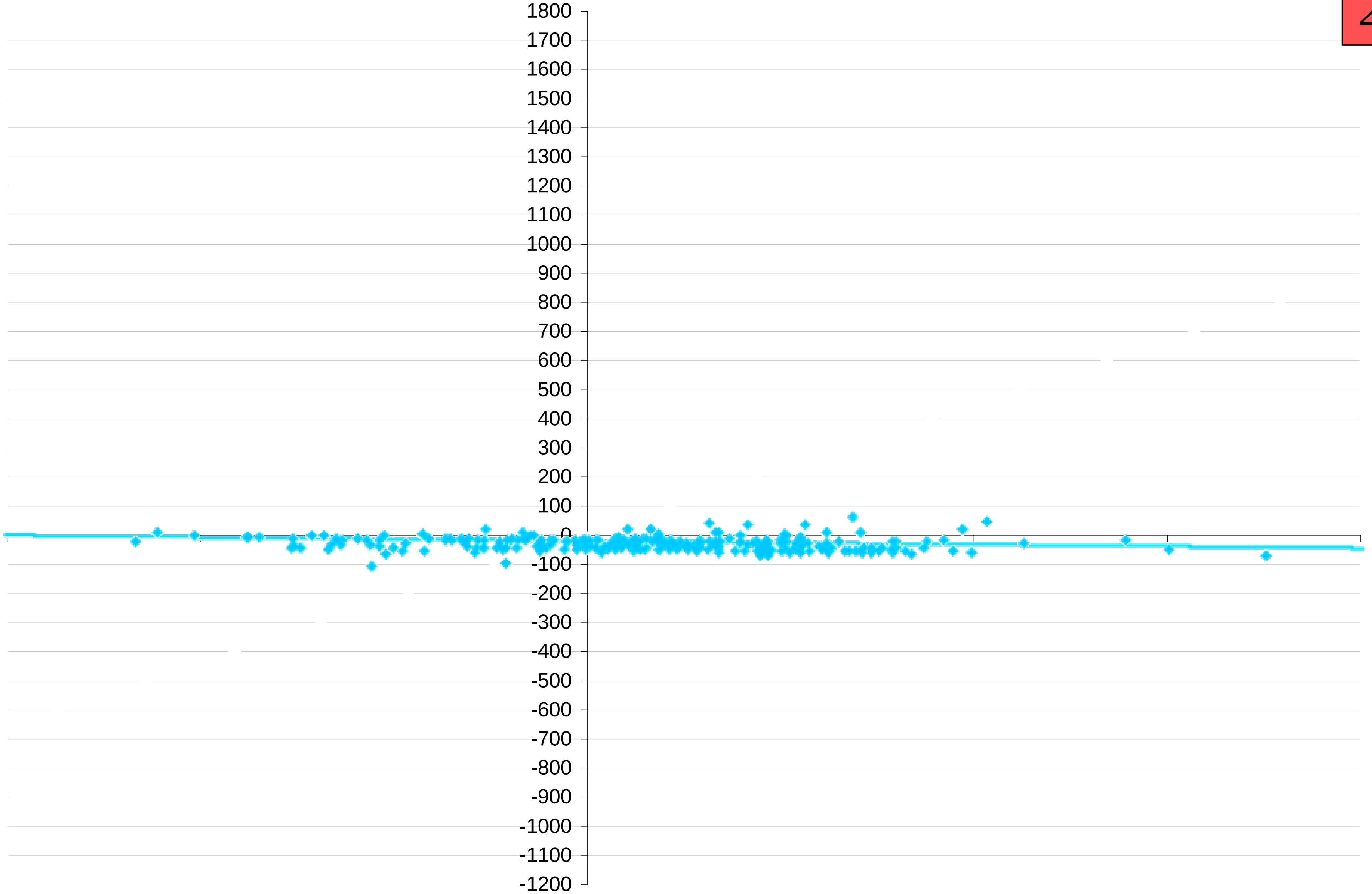
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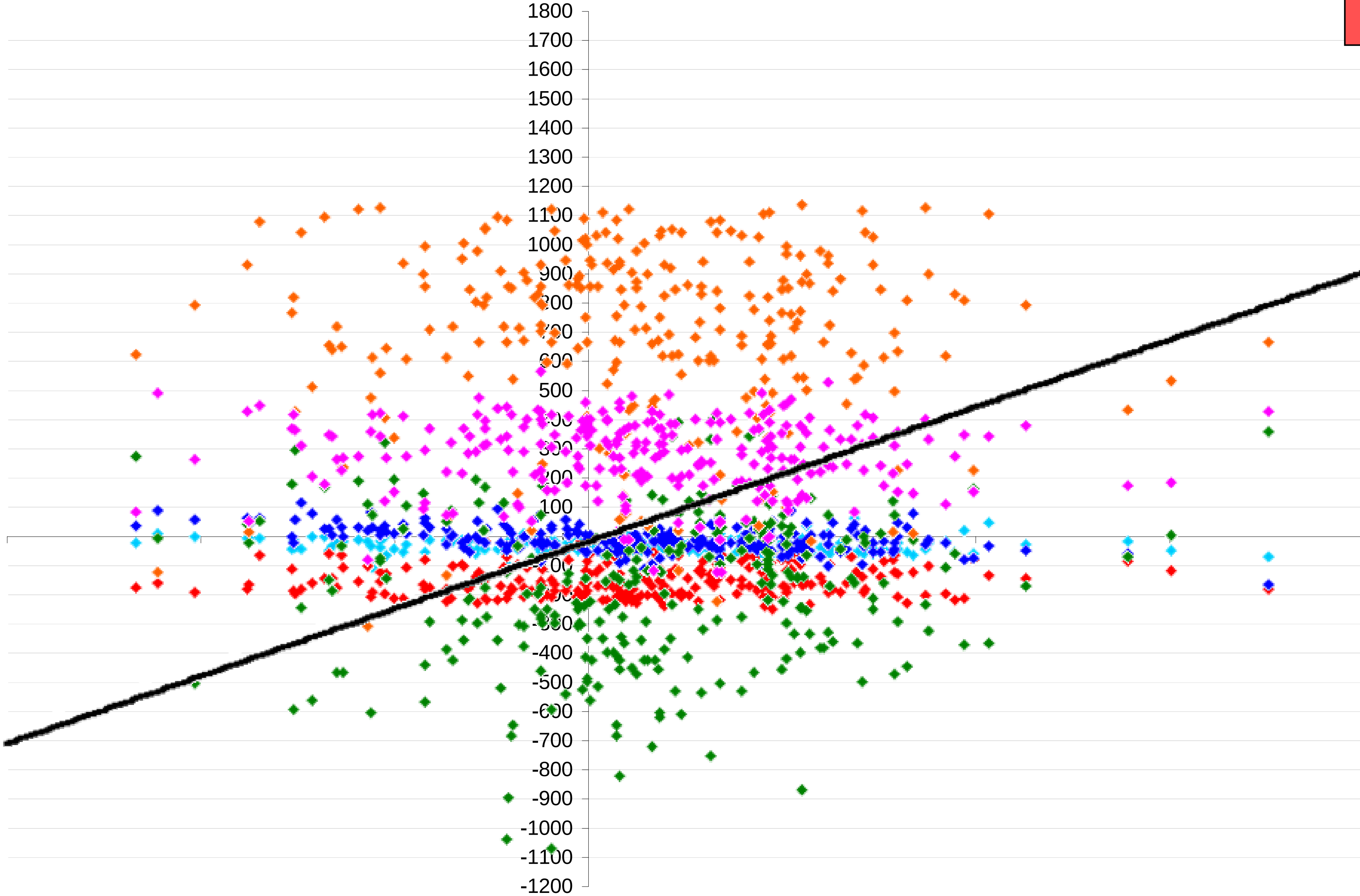
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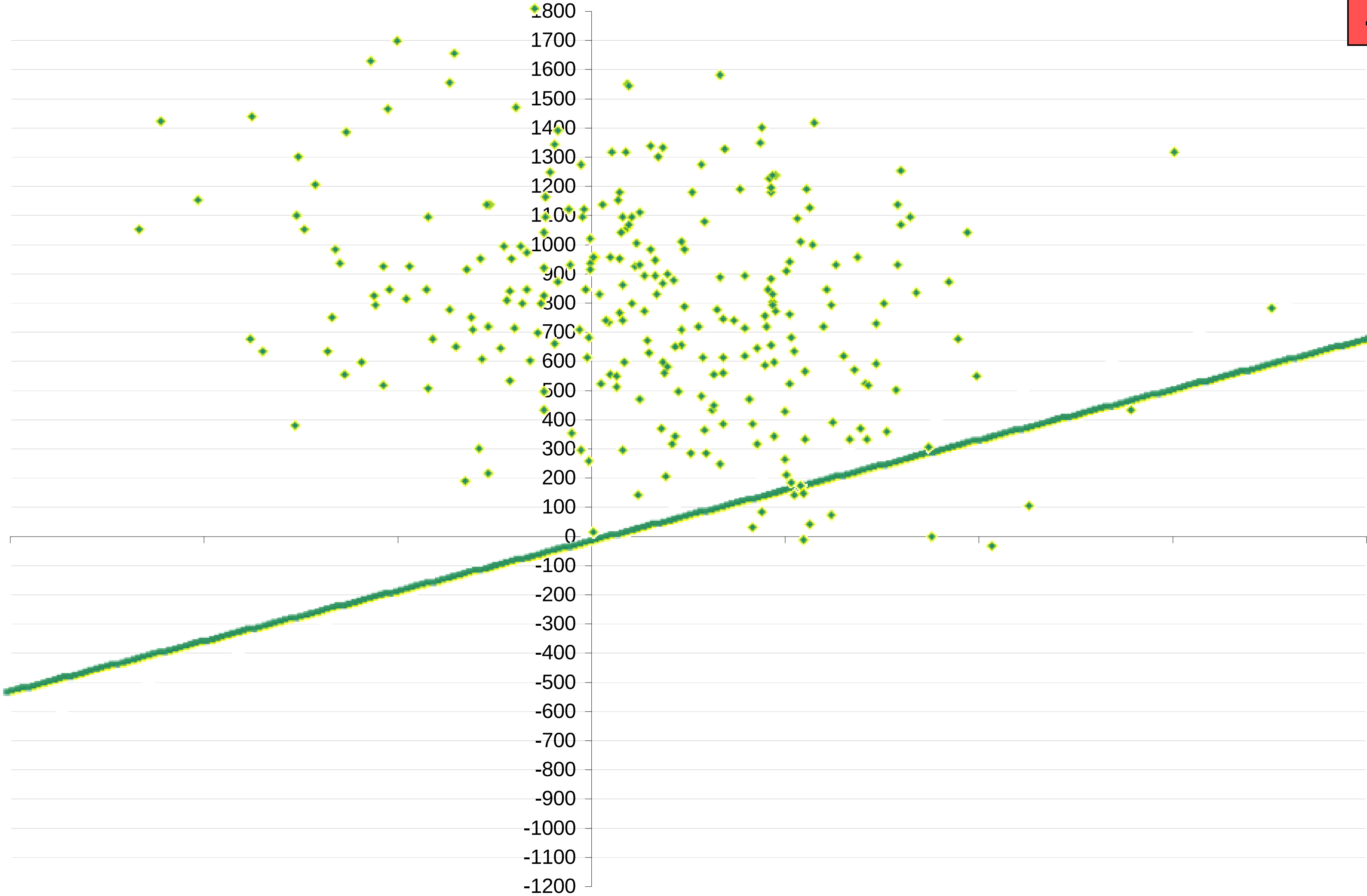
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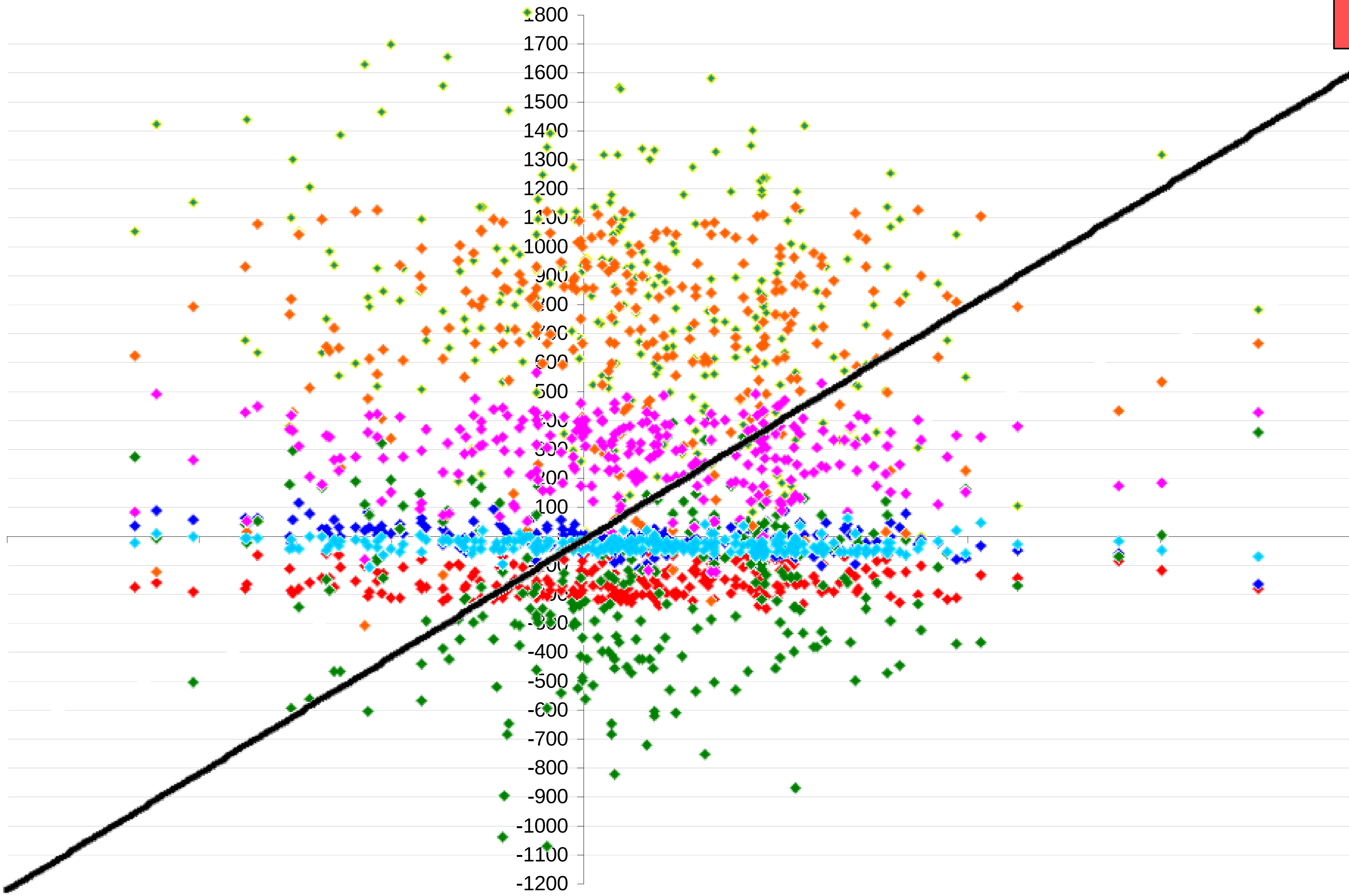
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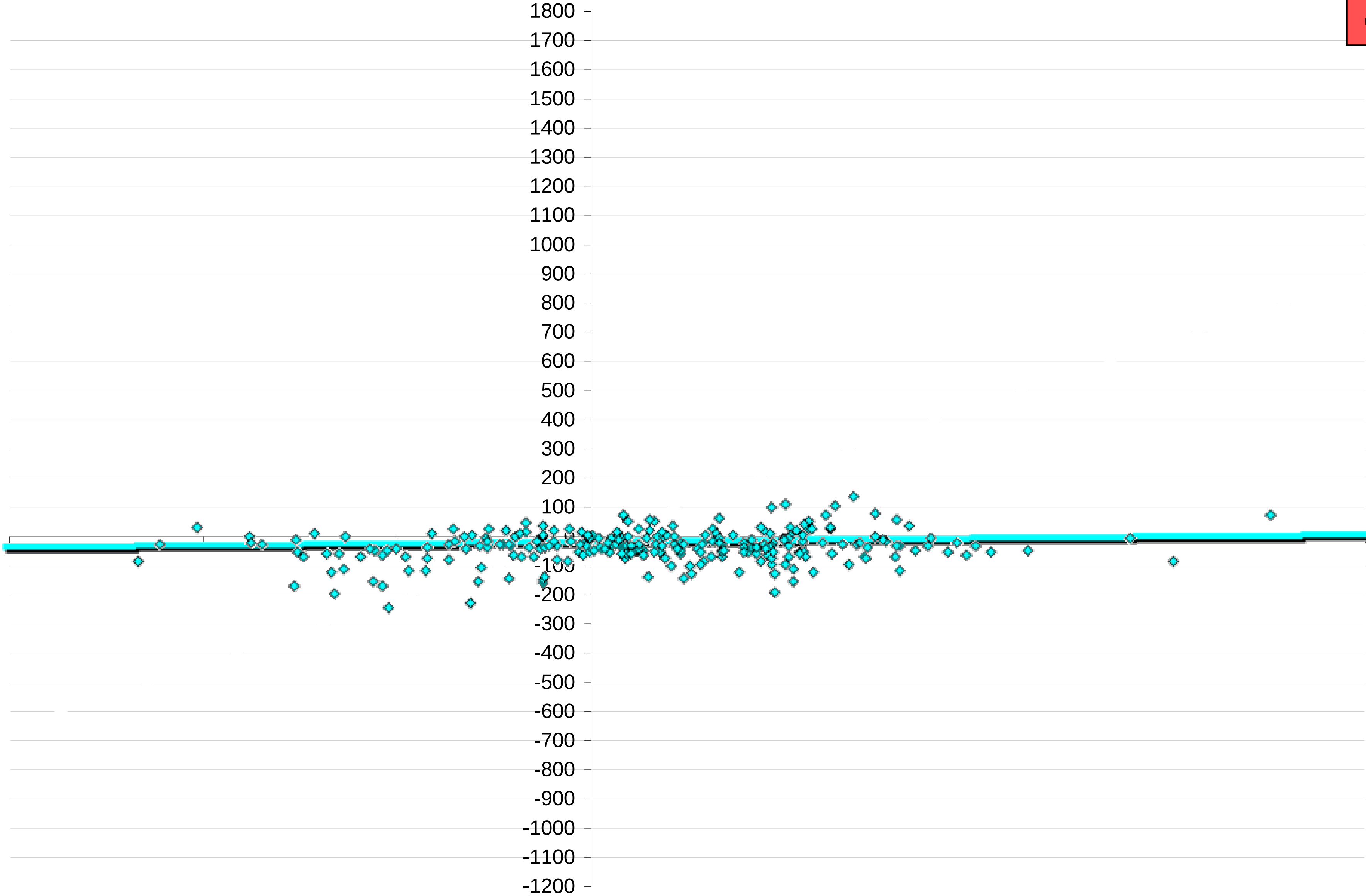
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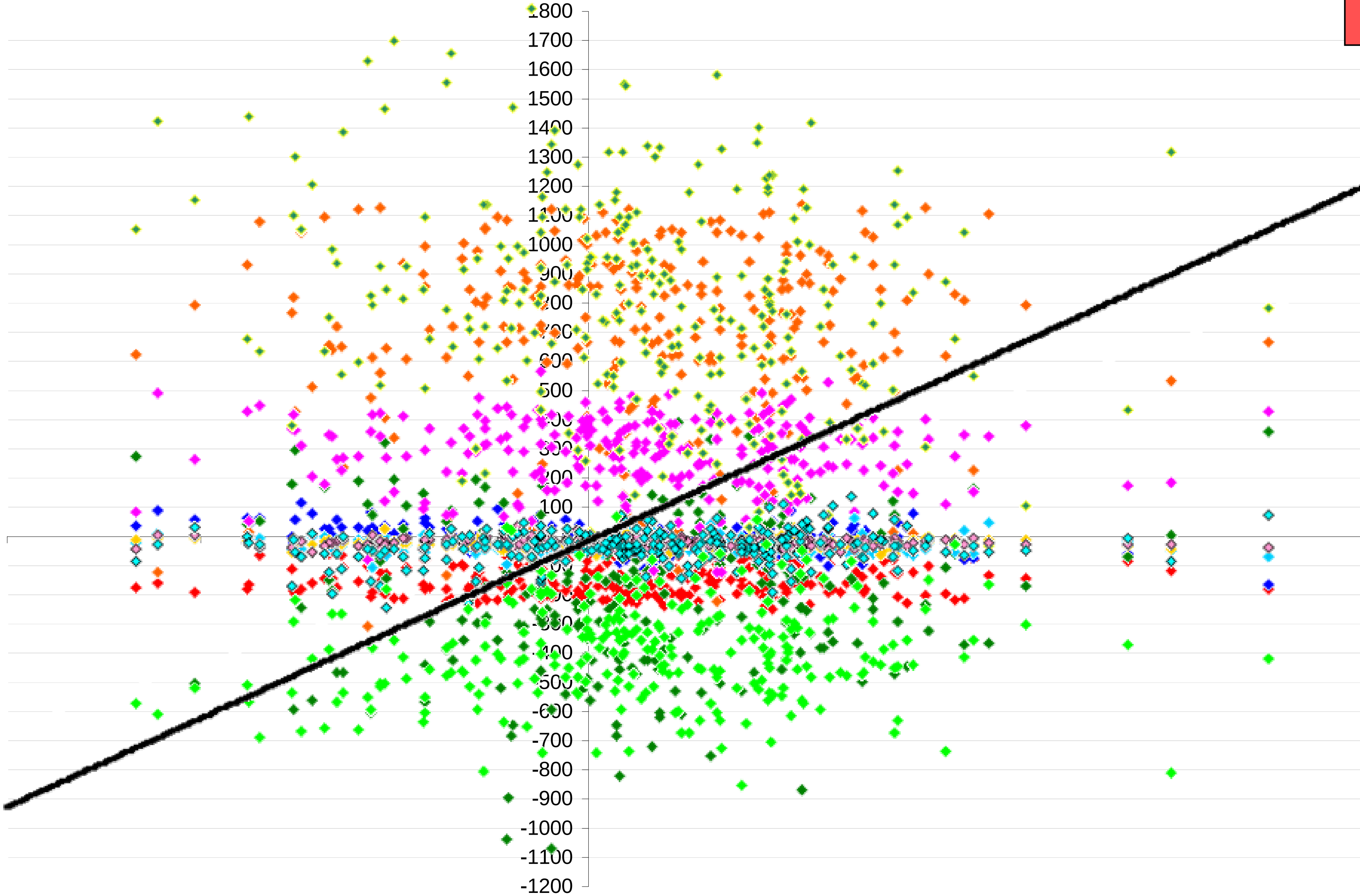
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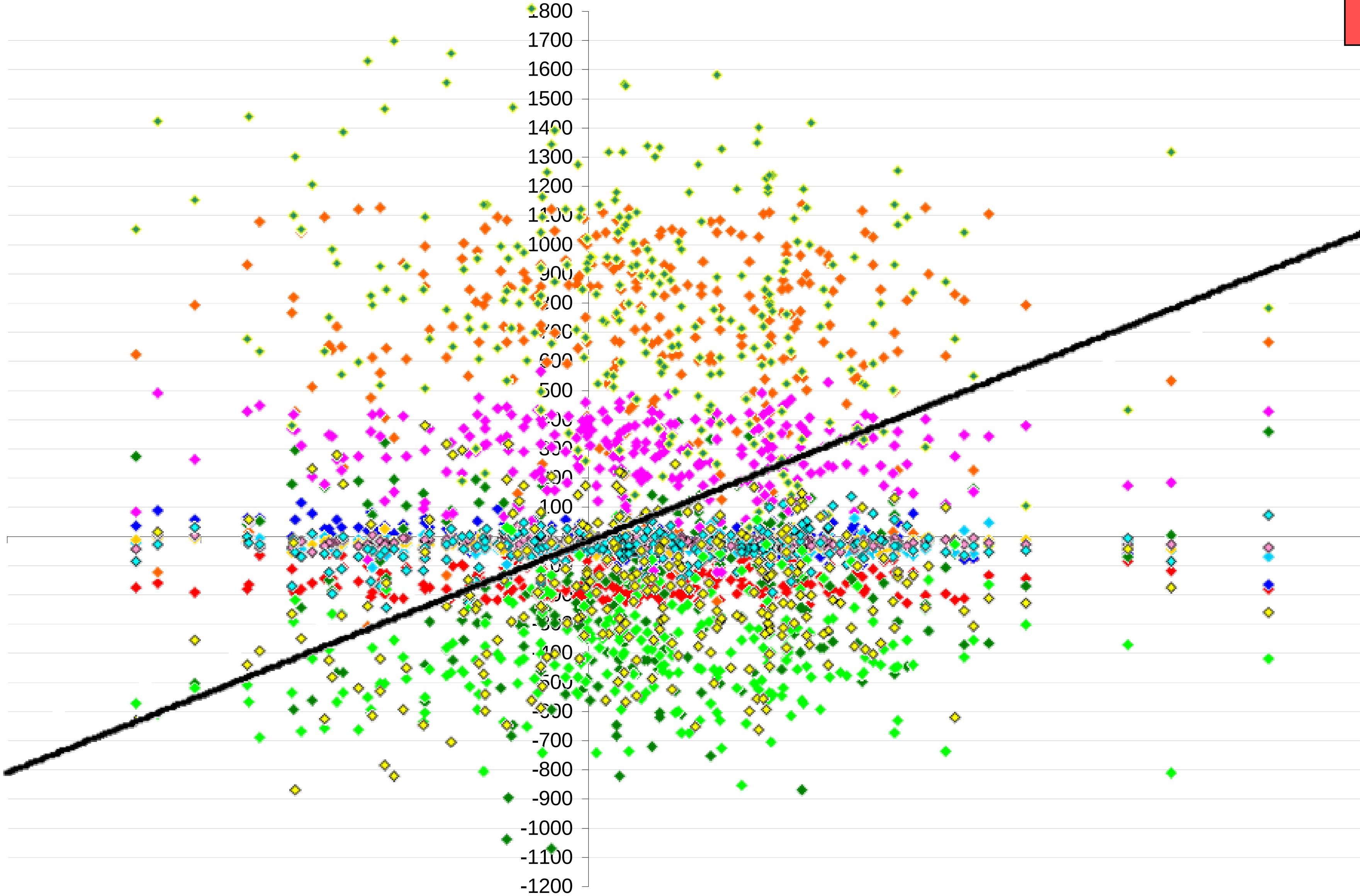
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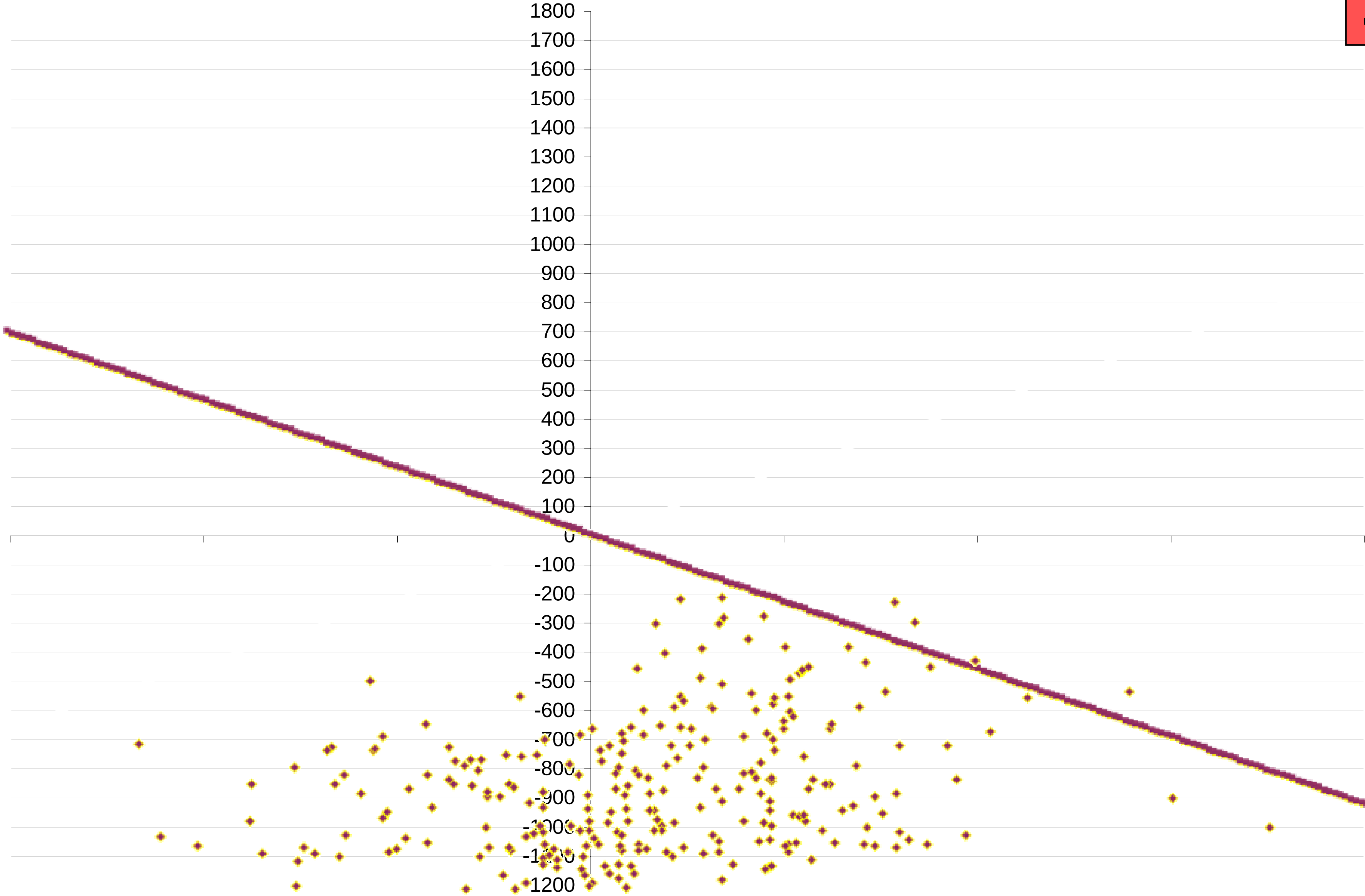
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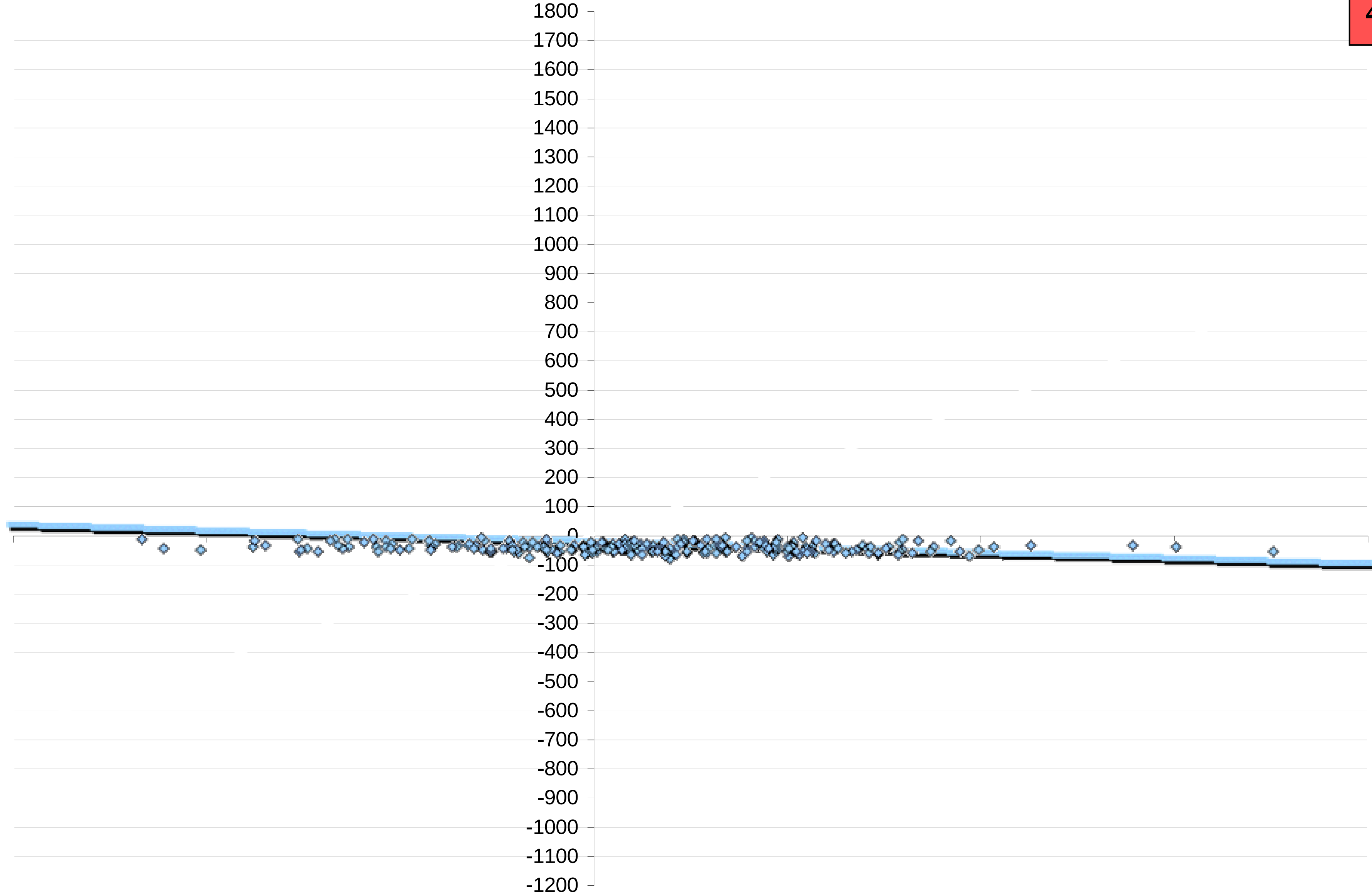
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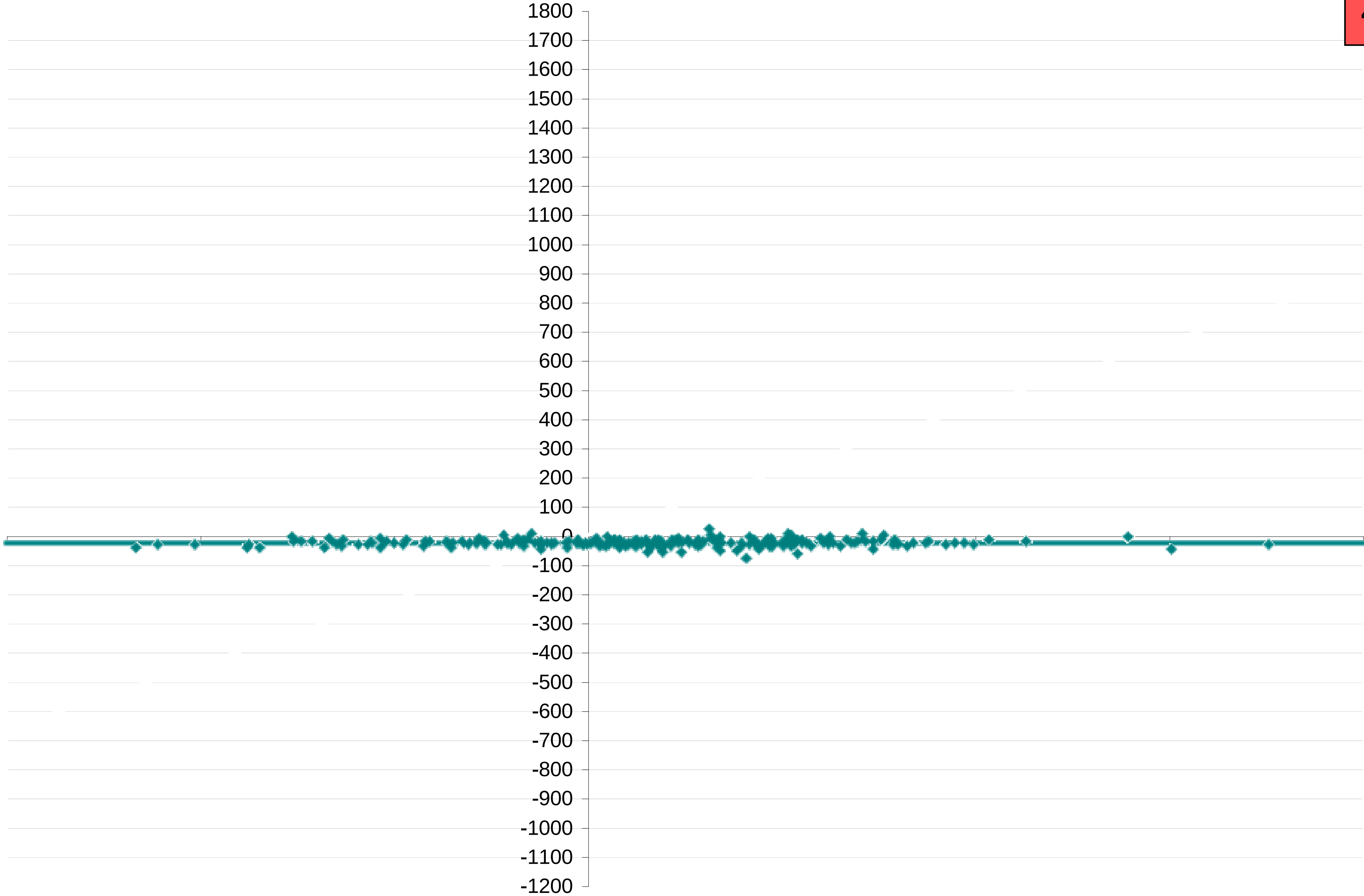
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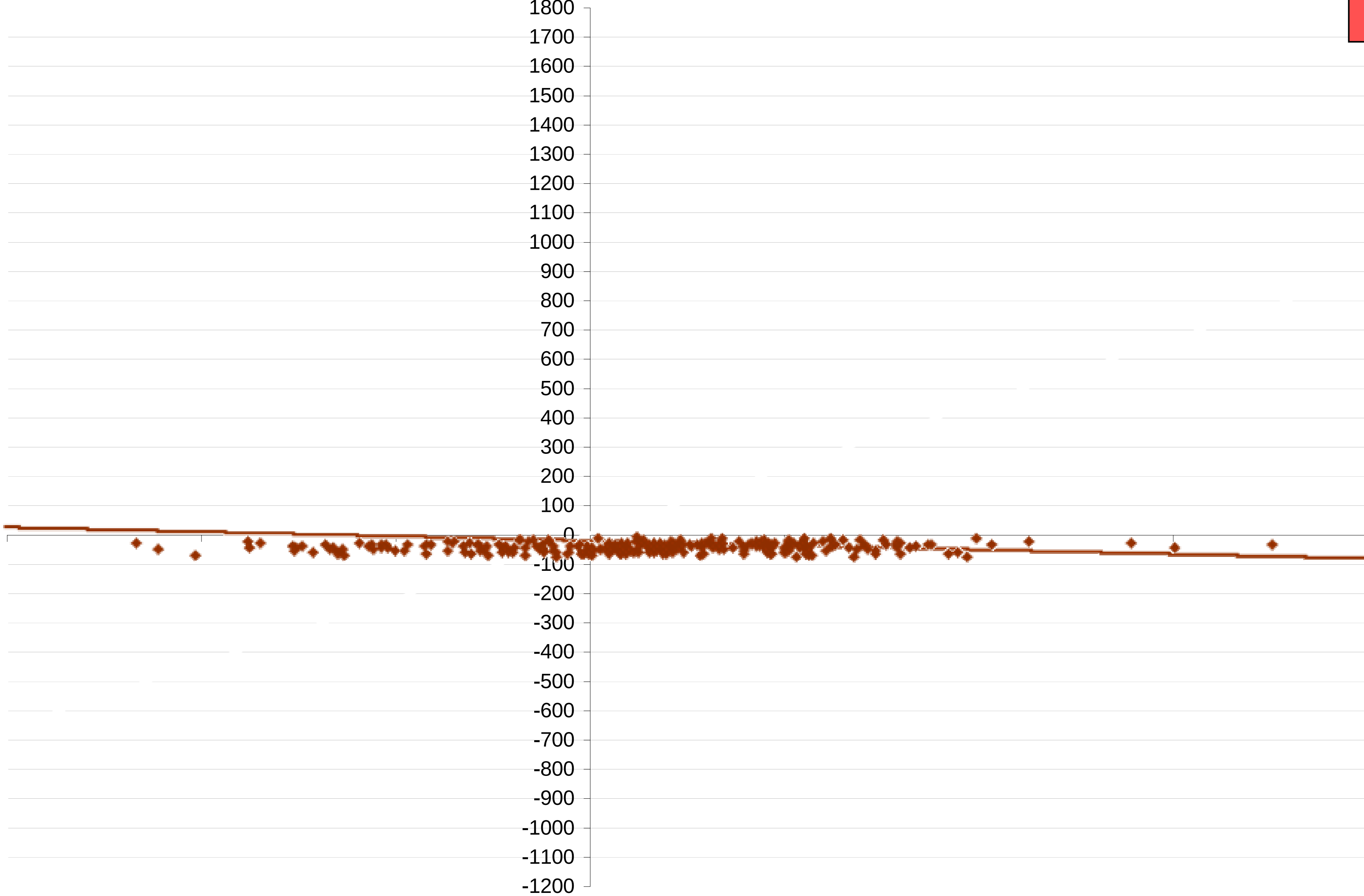
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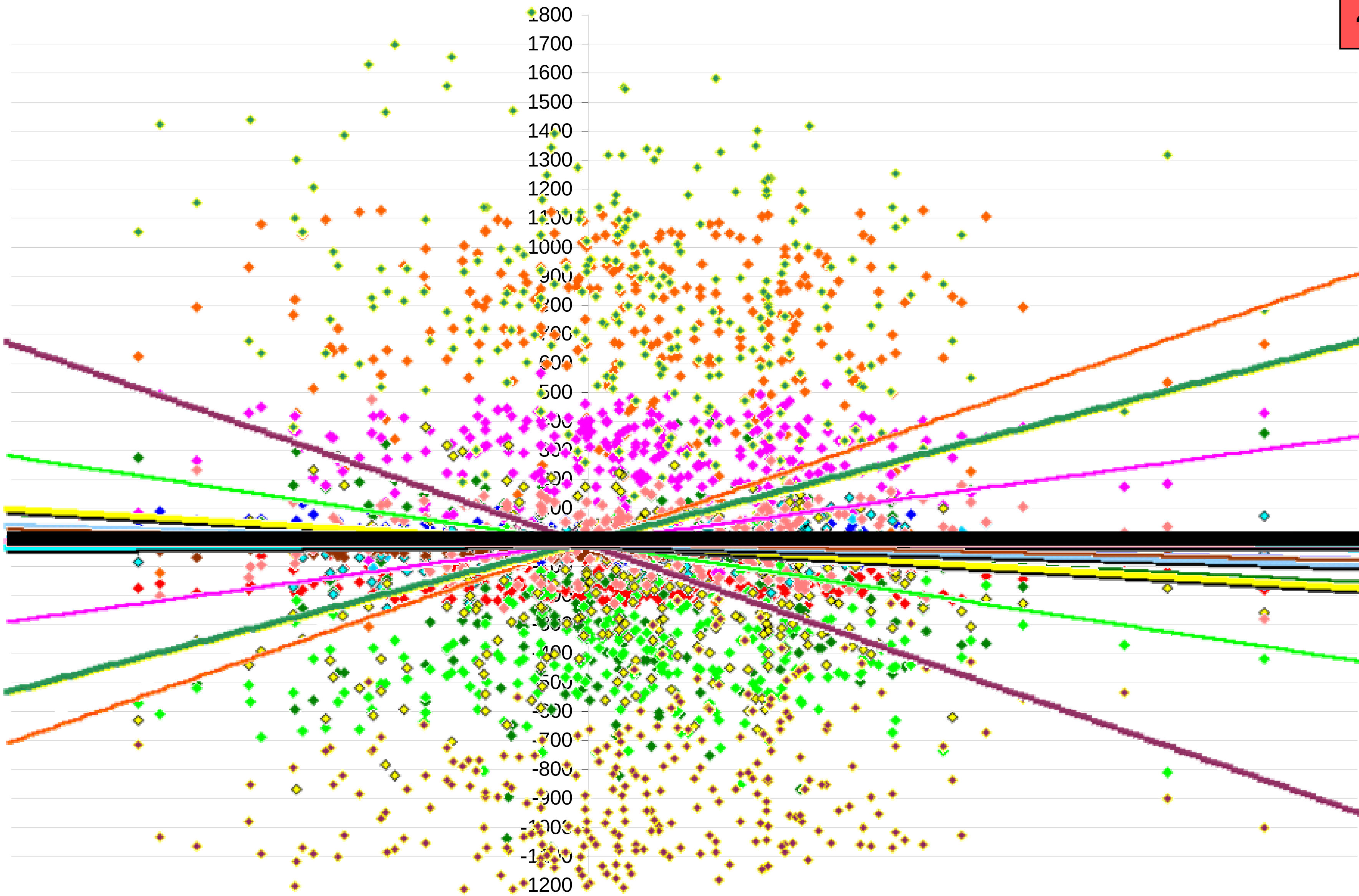
1800
1700
1600
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1400
1300
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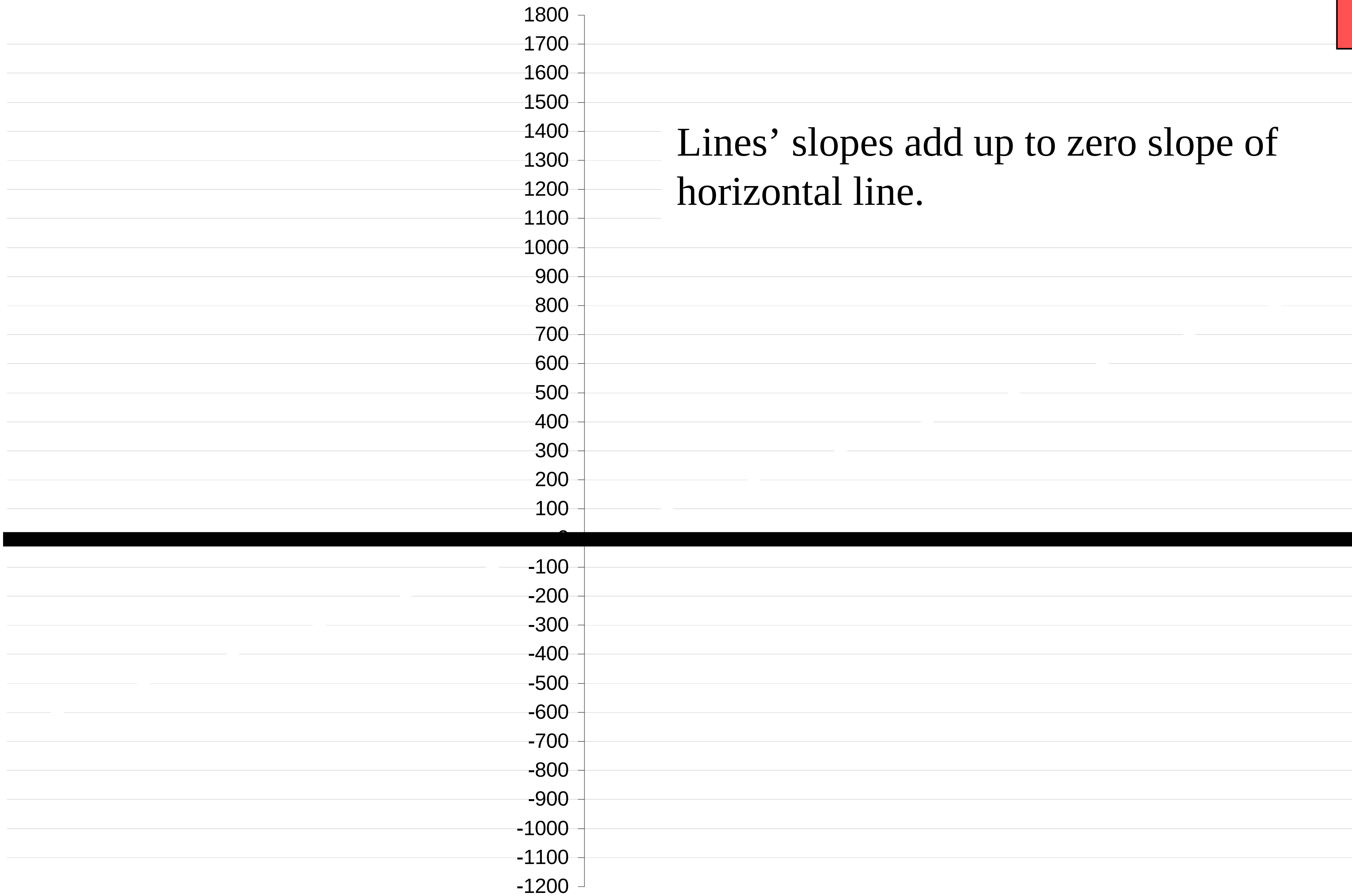
1000
500
-700
-900
-1200


1800
1700
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-700
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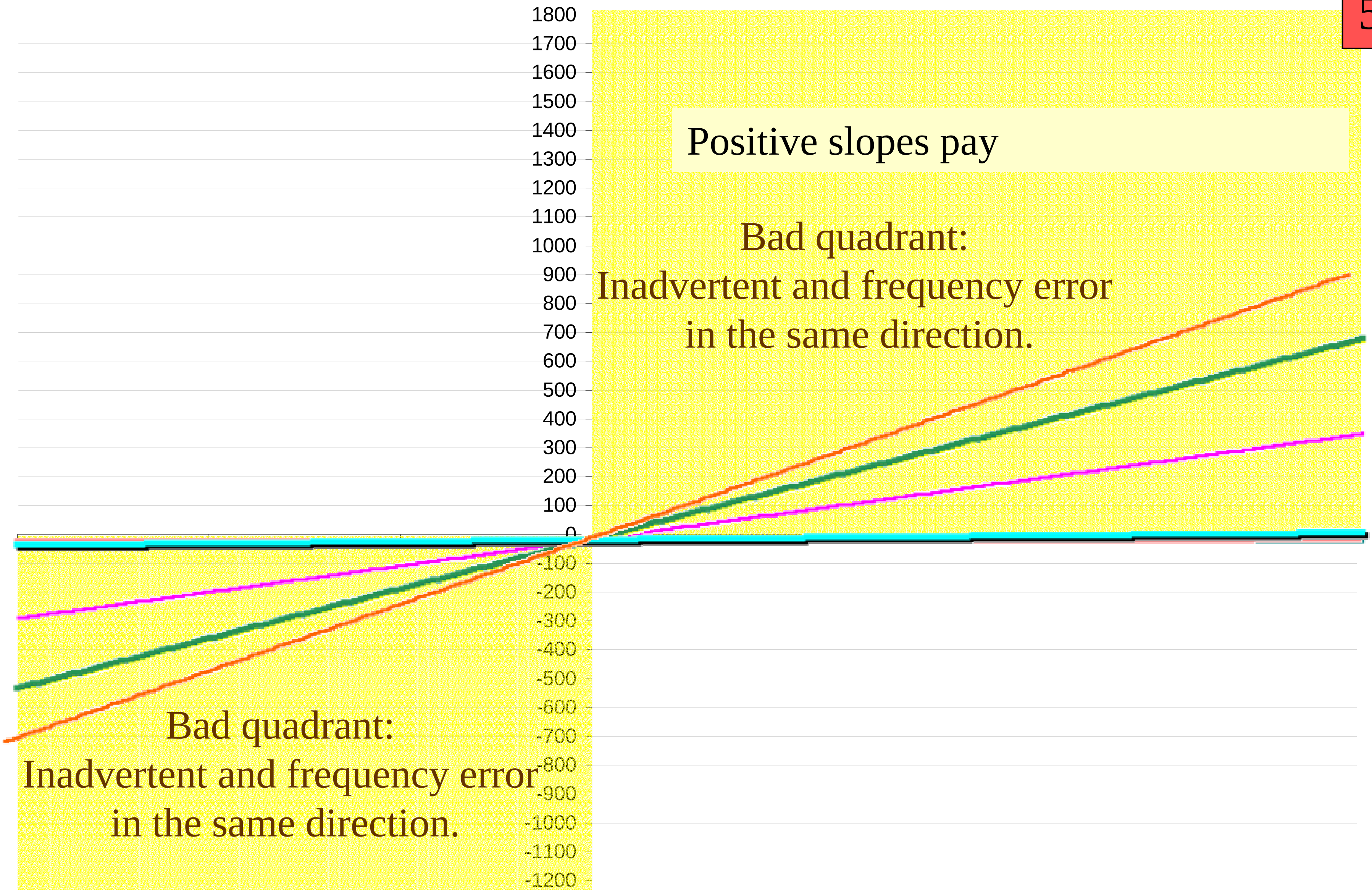
-1000
-1100
-1200
Lines’ slopes add up to zero slope of horizontal line.


1800
1700
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400
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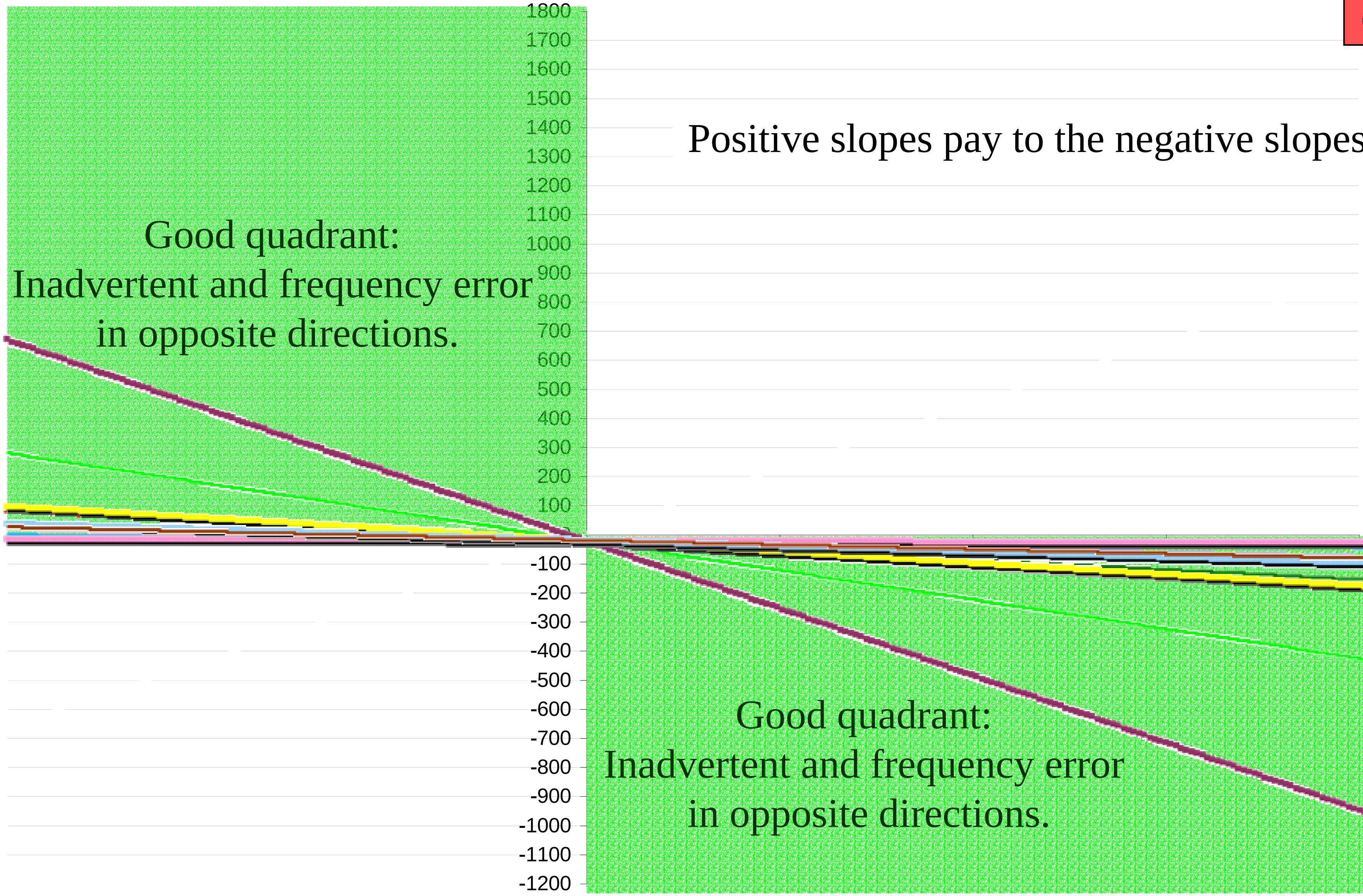
200
100
0
-100
-200
-300
-400
-500
-600
-700
-800
-900
-1000
-1100
-1200
Positive slopes pay
Bad quadrant:
Inadvertent and frequency error
in the same direction.
Bad quadrant:
Inadvertent and frequency error
in the same direction.


Positive slopes pay to the negative slopes
Good quadrant:
Inadvertent and frequency error
in opposite directions.
Good quadrant:
Inadvertent and frequency error
in opposite directions.
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
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200
100
-100
-200
-300
-400
-500
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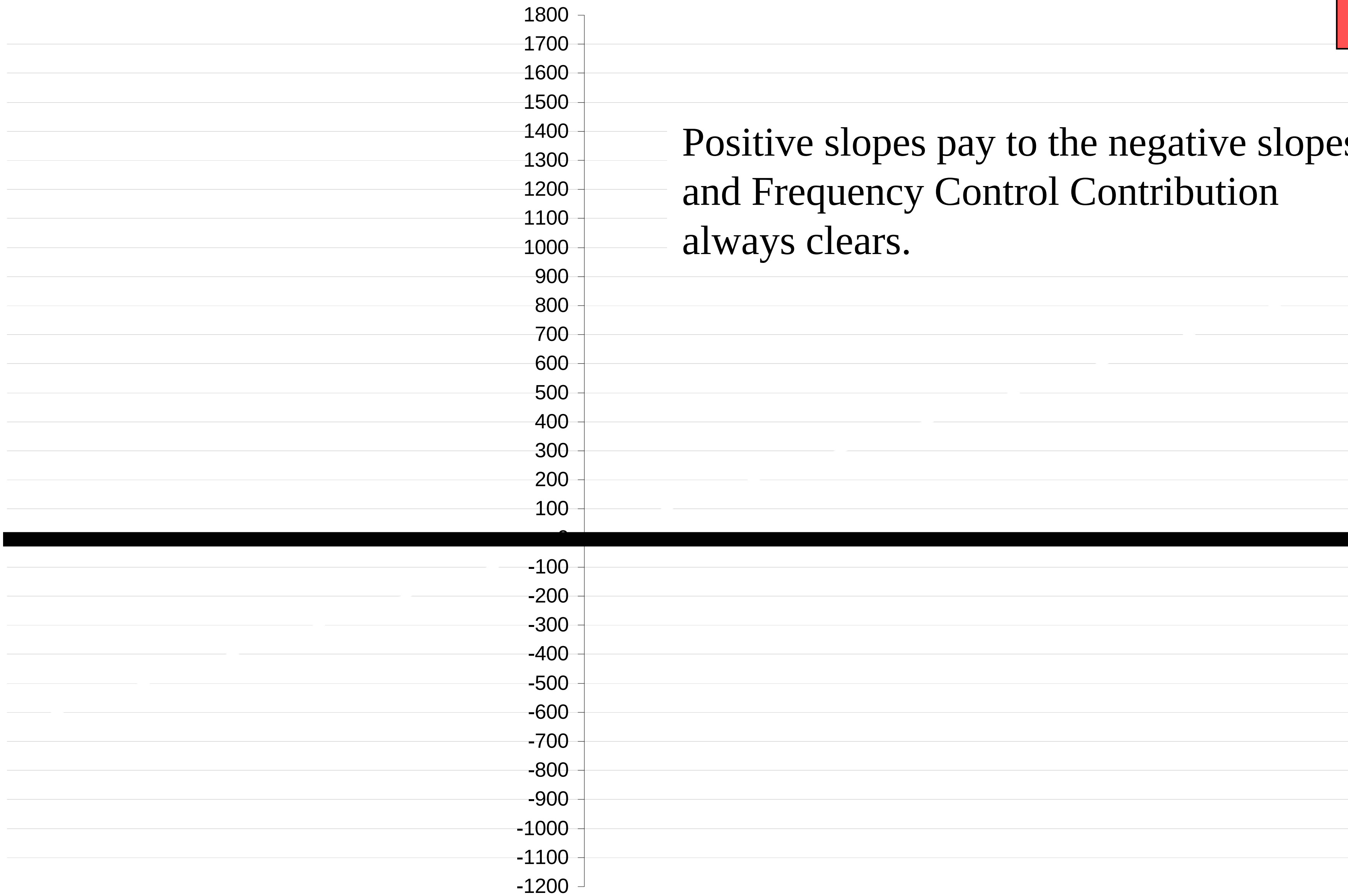
-700
-800
-900
-1000
-1100
-1200



Positive slopes pay to the negative slopes and Frequency Control Contribution always clears.

Market price of a distribution of points over time, not of a sharp point in time

- Real-time transactions cannot be done moment-by-moment deterministically/deliberately
  - Time is too short
- Real time performance must be managed, measured and valued as a statistical distribution
  - Classical physics versus quantum mechanics
    - joint-indeterminacy of position and momentum
  - Joint-indeterminacy of time-quantity and reliability-pricing
    - reliability pricing of a time average

“Computational Equivalence” (Wolfram/Mathematica): computational limitations in humans & physical nature

Tiered real-time market

- Three tiered market for frequency contribution
  - NERC, Balancing Authorities, local entities.
    - Frequency is a public good requiring an authority like NERC to drive the frequency-contribution markets by the threat of penalty.
      - This meets both a reliability and a markets objective
- Balancing Authorities must settle their FCC monthly
  - Since inadvertents sum to zero by definition, Balancing Authorities always clear

Tiered real-time market (cont.d)

- Balancing Authorities must also comply with CPS frequency targeting, acting as agents subject to NERC penalty
  - FCC does not target frequency
  - NERC CPS penalty will prompt Balancing Authorities to trade their CPS rights instead of paying the penalty, the way DOE pollution penalties prompted the market for pollution rights.
    - To meet their monthly CPS scores, Balancing Authorities will trade their frequency control contributions as an alternative to buying options on frequency support

Tiered real-time market (cont.d)

- FCC is open and scalable to a market below Balancing Authorities
  - Balancing Authorities can apply FCC to their constituent entities to incent entities’ self-provision and good performance, thereby minimizing the Balancing Authority’s own local intervention

Ancillary services markets

- Ancillary services markets need to be developed as robust options markets
  - The option price is driven by volatility which is another way to capture/express Frequency Control Contribution

Relation between CPS1

and

Frequency Control Contribution

$$FCC_{\overline{p}}$$

$FCC_{\bar{p}}$ extends to all inadvertent the price of the trading & procurement of residual inadvertent done to get compliant with $CPS1$.

The $CPS1$ tolerance band limits and drives $FCC_{\bar{p}}$.

$CPS1$ boils down to

$$\frac{\overline{\beta_{i,\,\Delta t}} - \overline{b_{i,\,\Delta t}}}{-\overline{b_{i,\,\Delta t}}} AVG\left(\overline{\Delta F}^{2}_{\Delta t}\right) \le \varepsilon^{2}$$

$$\left(\overline{\beta_{i,\,\Delta t}} - \overline{b_{i,\,\Delta t}}\right) AVG\left(\overline{\Delta F}^{2}_{\Delta t}\right) \le -\overline{b_{i,\,\Delta t}}\,\varepsilon^{2}$$

where $-\overline{b_{i,\,\Delta t}}$ is a least-squares estimator of bias $\overline{B_{i,\,\Delta t}}$

With bias $\overline{B_{i,\,\Delta t}} =_{set} 0$,
tolerance band $-\overline{b_{i,\,\Delta t}}\,\varepsilon^2 = 0$ and
$CPS1$ becomes

$$\overline{\beta_{i,\,\Delta t}}\,AVG\left(\overline{\Delta F_{\Delta t}}^{\,2}\right) \le 0$$

$$\frac{AVG(T_i \times \overline{\Delta F_{\Delta t}})}{AVG\left(\cancel{\overline{\Delta F_{\Delta t}}^{\,2}}\right)}\,\cancel{AVG\left(\overline{\Delta F_{\Delta t}}^{\,2}\right)} \le 0$$

$$AVG(T_i \times \overline{\Delta F_{\Delta t}}) \le 0$$

$FCC_{\overline{p}}$ monetization is equivalent to $CPS1$ without bias obligation/tolerance:

$$AVG(T_i \times \Delta F_{\Delta t}) \le 0.$$

$FCC_{\overline{p}}$ monetization makes this equation hold because

$$-k \times AVG(I_i \times \overline{\Delta F}_h) = FCC_{\overline{p}}$$

since

$$FCC_h \times p_{10\overline{\beta}} = -10\overline{\beta}_{i,h} \times p_{10\overline{\beta}} = -\frac{AVG\left(I_i \times \overline{\Delta F}_h\right)}{\cancel{AVG}\,\overline{\Delta F}_h^{2}} \times p_{10\overline{\beta}} = FCC_{\overline{p}}$$

and $p_{10\overline{\beta}} \overset{?}{\approx} k \times \cancel{AVG}\left(\overline{\Delta F}_h^{2}\right)$

Trading/procurement of $\overline{\beta_{i,h}}$ to get within the $CPS1$ tolerance band ε^2 drives the price $p_{10\overline{\beta}}$ of all $\overline{\beta}_{i,h}$

Approximate **CPS1**

On average over the past year:

π: Instantaneous Probability

$\bar{\varepsilon}$: Annual standard deviation of $\overline{\Delta F}$

Target RMS: $\varepsilon = \sqrt{\bar{\varepsilon}^2 + \mu^2\left(\overline{\Delta F}\right)}$

$\mu\left(\overline{\Delta F}\right)$: Year's Mean of $\overline{\Delta F}$

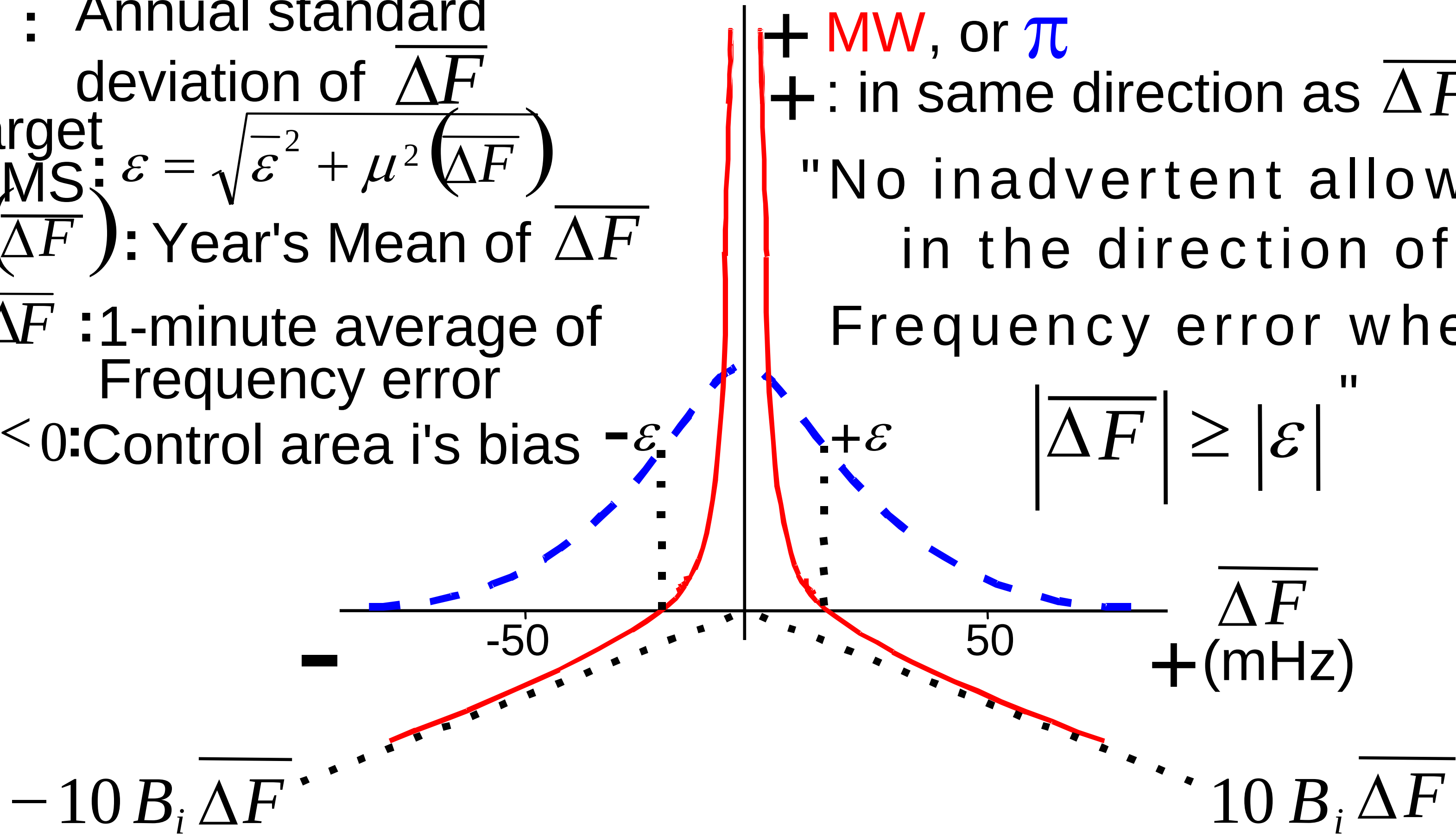
$\overline{\Delta F}$: 1-minute average of Frequency error

$B_i < 0$: Control area i's bias

+ MW, or π

+ : in same direction as $\overline{\Delta F}$

"No inadvertent allowed in the direction of Frequency error when $\left|\overline{\Delta F}\right| \geq |\varepsilon|$"

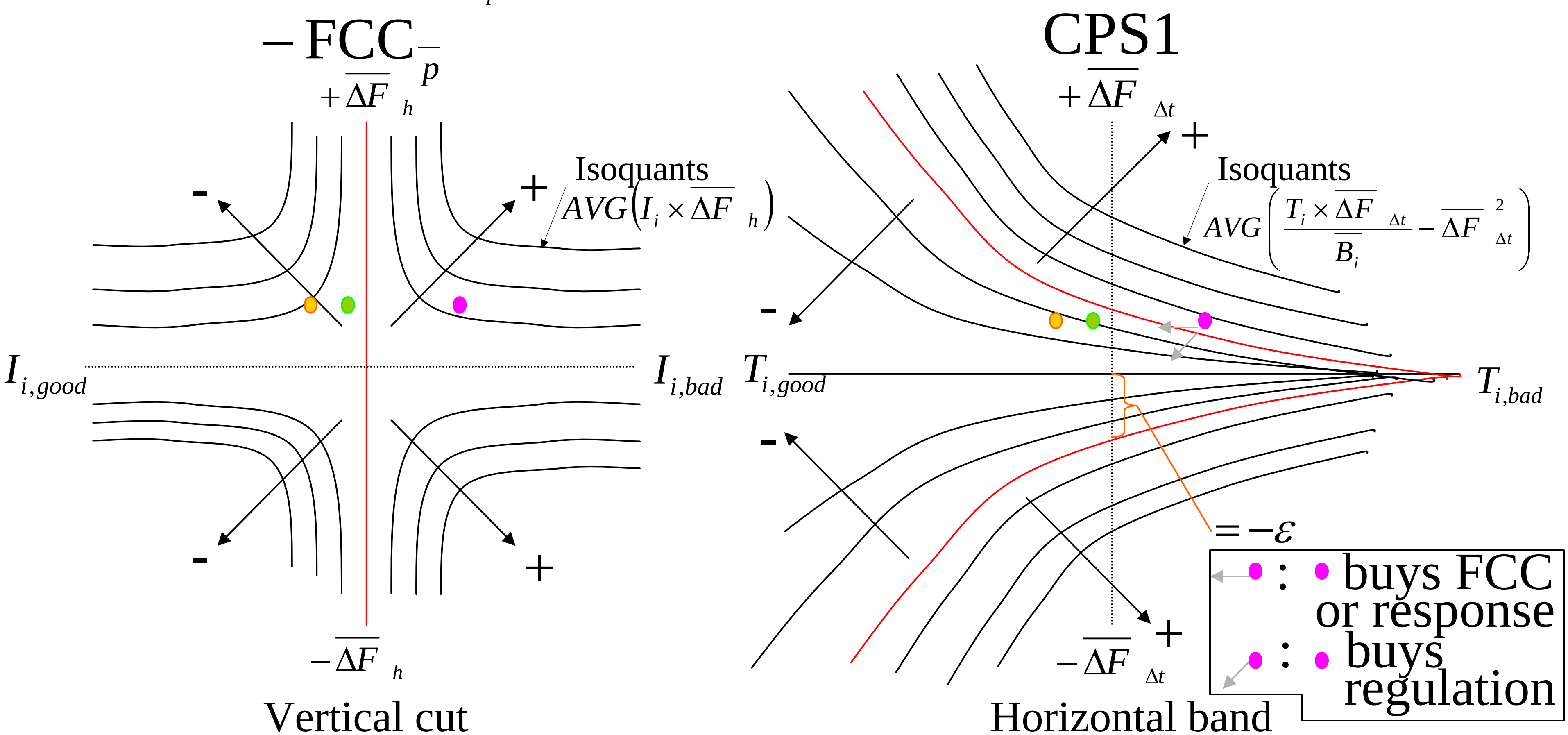


: Control area i's maximum allowed 1-minute average tie-line error (plus response obligation) in direction of the frequency error:

$$\overline{\Delta T_i} = -10 B_i \left(\varepsilon \frac{\varepsilon}{\overline{\Delta F}} - \overline{\Delta F} \right)$$

: One-year target probability density of 1-minute averages of frequency error, adjusted for deviation of the mean from 0

$FCC_{\bar{p}}$'s cut is perpendicular to CPS1's cut/band.
Payments for traded FCC_hs would sum to zero around CPS1's cut/band when $\left|\overline{\Delta F}_{\Delta t}\right| \le \varepsilon$.
There is excess demand for traded FCC_hs outside CPS1's cut/band when $\left|\overline{\Delta F}_{\Delta t}\right| > \varepsilon$
whence receipt of $FCC_{\bar{p}}$ reduces CPS1 penalty to incent BAs to get back inside.



$FCC_{\bar{p}}$'s vertical cut gets stretched right from the middle into CPS1's horizontal band.
BA • outside his CPS1 cut/band buys enough FCC_h from BAs • & • to get inside his CPS1 cut/band & avoid CPS1 penalty, and thereby helps set the FCC_h settlement price $p_{10\bar{\beta}}$.

To Get
Hourly Decomposition of
Frequency Control Contribution:

Interpret
Frequency Control Contribution as an 11-day average of 1-hour Frequency Control Contributions

FCC_h (11 day* average):
*264 hours

$$-\frac{AVG\left(I_i \times \overline{\Delta F}_h\right)}{AVG\,\overline{\Delta F}_h^2} = FCC_h$$

$$-AVG\frac{I_i \times \overline{\Delta F}_h}{AVG\,\overline{\Delta F}_h^2} = FCC_h$$

$$-AVG\frac{I_i \times \overline{\Delta F}_h}{(1/264)\sum_{h=1}^{264}\overline{\Delta F}_h^2} = FCC_h$$

FCC_h (hour 1):

$$-\frac{I_i \times \overline{\Delta F}_1}{(1/264)\sum_{h=1}^{264}\overline{\Delta F}_h^2} = FCC_1$$

(Column K of spreadsheet)

$$-\frac{264 \times I_i \times \overline{\Delta F}_1}{\sum_{h=1}^{264}\overline{\Delta F}_h^2} = FCC_1$$

1-hour FCC_h sum to 0 across control areas *i*
because I_i sum to 0 across the *i*'s.

$$\sum_{i=1}^{17} FCC_1 = -\sum_{i=1}^{17} \frac{I_i \times \overline{\Delta F}_1}{(1/264)\sum_{h=1}^{264} \overline{\Delta F}_h^{\,2}}$$

$$= -\frac{\sum_{i=1}^{17}\left(I_i \times \overline{\Delta F}_1\right)}{(1/264)\sum_{h=1}^{264} \overline{\Delta F}_h^{\,2}}$$

$$= -\frac{\overbrace{\left(\sum_{i=1}^{17} I_i\right)}^{=0} \times \overline{\Delta F}_1}{(1/264)\sum_{h=1}^{264} \overline{\Delta F}_h^{\,2}}$$

$$= 0 \quad \text{since} \quad \sum_{i=1}^{17} I_i = 0$$

Because 1-hour FCC_h sum to 0 across the control areas *i*,
the 264-hour averages of 1-hour FCC_h sum to 0 across the *i*'s.

$$
\begin{array}{lccccccc}
\sum_{i=1}^{17} FCC_{1,i} & = & FCC_{1,1} & + & FCC_{1,2} & + \cdots + & FCC_{1,17} & = 0 \\
 & & + & & + & & + & \\
\sum_{i=1}^{17} FCC_{2,i} & = & FCC_{2,1} & + & FCC_{2,2} & + \cdots + & FCC_{2,17} & = 0 \\
 & & + & & + & & + & \\
 & \vdots & \vdots & & \vdots & & \vdots & \vdots \\
 & & + & & + & & + & \\
\sum_{i=1}^{17} FCC_{264,i} & = & FCC_{264,1} & + & FCC_{264,2} & + \cdots + & FCC_{264,17} & = 0
\end{array}
$$

$$
(1/264)\times\{\text{column } 1\},\quad (1/264)\times\{\text{column } 2\},\quad \ldots,\quad (1/264)\times\{\text{column } 17\}
$$

$$
\sum_{i=1}^{17} FCC_{h,i} = FCC_{h,1} + FCC_{h,2} + \cdots + FCC_{h,17} = 0
$$

Marginal FCC Cost
of
Inadvertent
as
Amount of
Frequency Control Contribution per
MWh of Inadvertent

FCC_h (hour 1):

$$-\frac{264 \times I_i \times \overline{\Delta F}_1}{\sum_{h=1}^{264} \overline{\Delta F}_h^{\,2}} = FCC_1$$

Marginal FCC cost (in units of FCC_1) of Inadvertent I_i:

(Column F of spreadsheet)

$$-\frac{264 \times \overline{\Delta F}_1}{\sum_{h=1}^{264} \overline{\Delta F}_h^{\,2}} = FCC_1 MC_{I_i}$$

To avoid economic gaming
a continuous
Frequency Control Contribution value of
Inadvertent Interchange
is needed because the
hourly Frequency Control Contribution
of Inadvertent Interchange
is often very big relative to the
Energy Component of
Inadvertent Interchange.

Inadvertent Payments by/to (Green) Control Area 3
Assuming price of 10¢/MWh/Hz of FCC

	For Energy +	For FCC = $FCC_t \times$ 10¢/MWh/Hz	Net
Day 1: Hour 4	Receive $950 @$10/MWh	Pay $481 @ $5.06/MWh $=FCC_4 MC_{I_3}$	Receive $469
Day 1: Hour 11	Pay $722 @$20/MWh	Receive $4314 @ $11.95/MWh $=FCC_{12} MC_{I_3}$	Receive $3592
Day 6: Hour 2	Receive $1350 @$10/MWh	Pay $2324 @ $17.21/MWh $=FCC_{146} MC_{I_3}$	Pay $974
Day 6: Hour 12	Pay $2420 @$20/MWh	Pay $1767 @ $14.60/MWh $=FCC_{156} MC_{I_3}$	Pay $4197
11-day Average (Row 277 Column X of spreadsheet)		$2.06/MWh	